Design and Analysis of Fault Diagnosis and Fault-tolerant Control for a Class of MIMO Nonlinear State Systems

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Abstract: This paper presents a fault diagnosis and fault-tolerant control algorithm, which can be used for a class of multi-input multi-output (MIMO) nonlinear state systems. First, a state estimator is proposed, which is able to detect fault occurrence, by using a residual signal. Second, when the state is at an abnormal condition, the fault-tolerant control will be triggered to minimize the impact of the fault occurrence. This fault-tolerant control is designed by using a robust controller (original controller), and an on-line approximator to capture a nonlinear function that indicates the fault occurrence. The detailed analysis is given for the proposed fault accommodation control.

Key words: Fault Diagnosis, Fault Detection, Nonlinear Observer, On-line Approximator, Fault-tolerant Control.

1 Introduction

Over the past two decades, there has been an increasing demand for the control systems to be safer and more reliable. Therefore, fault diagnosis (or fault monitoring) has been becoming an important topic in control research and practical applications. Fault diagnosis has been studied intensively. For example, Visinsky et al. [1] present an expert system model for handling fault detection. The model based on fault diagnosis has been widely accepted in computer control due to the availability of input/output data [2]. Some results regarding model-based fault diagnosis can be found in the literature [3-9]. For example, Wang and Daley [3] present an adaptive estimator algorithm for diagnosing actuator faults; Wunnenberg and Frank [4] propose a mathematical model to diagnose incipient faults. However, model-based fault detection algorithms depend on the assumption that an accurate mathematical model is available. In real situations, that may not be practical, for sometimes it is quite difficult to develop an accurate model. By introducing neural networks, an adaptive fault diagnosis algorithm has been presented in the work of [5, 6], while the paper applies a neural network to approximate the unknown fault. Moreover, in many practical applications, it is necessary not only to diagnose but also to accommodate any faults as quickly as possible, this is called fault-tolerant control. Several types of fault-tolerant control have been reported. For example, Visinsky et al. [1] present an expert rule-based system for achieving fault-tolerant control; Tao et al. [7] present an adaptive method for accommodating actuator faults; Polycarpou and Helmic [9] present a fault diagnosis and fault-tolerant control approach; Diao and Passino [8] develop a stable adaptive controller to implement fault-tolerant engine control; Zhang et al. [10] present an information-based fault-tolerant controller and discuss the detectability and stability issues of it. Unfortunately, these results [8-10] are based on single-input single output (SISO) systems. Research on multi-input multi-output (MIMO) systems have been carried out in recent years. For example, Chen et al. [12] develop an adaptive actuator failure compensator for a class of linear multivariable systems; Farrell et al. [11] present a learning method for fault accommodation control; and Visinsky et al. [13] present a dynamic fault accommodation control, which is applied to robotic systems. However, stability analysis has not been addressed [11, 13]. Furthermore, Huang et al. [14]

propose an information-based fault-tolerant control approach, which can be used in the presence of state and sensor failures, while Polycarpou [15] proposes a fault accommodation control scheme for a class of MIMO nonlinear systems. However, the detectability and multi-state stability issues are not addressed in the work of Huang et al. [14], while the work of Polycarpou [15] hinges on the assumption that the original controller (without considering faults) is based on a nominal model that is known a priori. The latest developments in MIMO systems can be found in the literature [16-18]. Jin et al. [16] design a passive fault fault-tolerant controller for a class of MIMO linear systems. However, their work cannot be applied to nonlinear systems. Nasiri et al. [17] present a passive actuator fault tolerant controller for a class of MIMO nonlinear systems. However, they do not consider the fault diagnosis, and this may result in the waste of control energy. Mbarek and Bouzrara [18] propose a fault-tolerant controller based on multiple models, which are approximated by a set of linear, time-invariant and causal sub-models. However, this approach cannot handle uncertainties that are unstructured but bounded. In addition, the detectability is not addressed in the work of Mbarek and Bouzrara [18].

The present paper investigates a class of uncertain MIMO nonlinear systems and designs a fault diagnosis and fault-tolerant control scheme. Here, it is assumed that the faults are unknown. Thus, a fault diagnosis algorithm based on a nonlinear observer, is developed. It is expected that the observed states will indicate significant deviation from the nominal values of the observer. The present fault accommodation scheme consists of an original controller, and a reconfigured controller that is used after fault detection. The present paper makes three main contributions. First, the original controller is robust against modeling uncertainties. This will guarantee satisfactory tracking performance (with a constant bound) in a normal operating mode. Second, the detectability and stability of the fault-tolerant control algorithm are addressed in detail. Third, the matching condition [15] is removed completely. Finally, simulation results are obtained to demonstrate goods performance of the proposed fault diagnosis and fault-tolerant control scheme.

The present paper is organized into six sections. Section 2 gives the problem background and system description. Fault diagnosis and detectability analysis are discussed in Section 3. Fault-tolerant control and stability analysis while considering multi-states are given in Section 4. A case study is given in Section 5. The concluding comments are presented in Section 6.

2 Backgrounds

This section presents the systems considered and the objectives of the present work. The MIMO nonlinear system is described by,

$$\left.\begin{aligned} x_i^{(n_i)} &= f_i(x,t) + \sum_{j=1}^{m} g_{ij}(x,t)u_j \\ &\quad + \eta_i(x,t) + \beta_i(t-T)\zeta_i(x) \\ y_i &= x_i \end{aligned}\right\}, \quad (1)$$

where

$x_i^{(n_i)} = d^{n_i}x_i/dt^{n_i}$,

$x = [x_1, \dots x_1^{(n_1-1)}, x_2, \dots x_2^{(n_2-1)} \dots x_m \dots x_m^{(n_m-1)}]^T$

$n_1 + n_2 + \dots + n_m = n$,

represent the system state, $u_j, j = 1,2,\dots,m$, represent the control signals, $y_i, i = 1,2,\dots,m$, define the system outputs, $f_i, g_{ij}, i,j = 1,2,\dots,m$, represent the known nonlinear functions, $\eta_i(x,t), i = 1,2,\dots, m$, define the uncertain terms, and $\beta_i(t-T)\zeta_i(x)$, $i = 1,2,\dots,m$, denote the function of fault occurrence.

Equation (1) can be re-arranged into the following form:

$$x^{(n)} = F(x,t) + G(x,t)u + \eta(x,t) + B(t-T)\zeta(x), \quad (2)$$

where

$x^{(n)} = [x_1^{(n_1)}, x_2^{(n_2)}, \dots, x_m^{(n_m)}]^T$,

$F(x,t) = [f_1(x,t), f_2(x,t), \dots, f_m(x,t)]^T$,

$$G(x,t) = \begin{bmatrix} g_{11}(x,t) & \dots & g_{1m}(x,t) \\ \vdots & \dots & \vdots \\ g_{m1}(x,t) & \dots & g_{mm}(x,t) \end{bmatrix},$$

$u = [u_1, u_2, \ldots, u_m]^T$,

$\eta(x,t) = [\eta_1(x,t), \eta_2(x,t), \ldots, \eta_m(x,t)]^T$,

$B(t-T) = diag\{\beta_1(t-T), \beta_2(t-T), \ldots, \beta_m(t-T)\}$,

$\zeta(x) = [\zeta_1(x), \zeta_2(x), \ldots, \zeta_m(x)]^T$.

The proposed system can represent a class of robotic systems; for example, mobile robots and n-link robots (see Fig.1).

(a) (b)

Fig. 1 Robotic systems: (a) n-link robot; (b) mobile robot.

Each fault is modeled with a time profile,

$$\beta_i(t-T) = \begin{cases} 1 - e^{-\theta_i(t-T)} & time\ \ t \geq T \\ 0 & time\ \ t < T \end{cases} \tag{3}$$

where θ_i is a positive constant, which represents a change rate of an actuator fault. It should be noticed that in (1), β_i represents whether the occurrence speed of a fault is fast or slow, while $\zeta_i(x)$ represents what the fault features are.

The present work has two main objectives. The first objective is to detect fault occurrence. The second objective is to accommodate the detected fault and maintain a stable closed-loop control.

Some assumptions are made now:

A1) $\zeta(x)$ is required to be uniformly continuous.

A2) $G(x,t)$ is required to be inverstible.

A3) $\eta_i(x,t)$ is required to be bounded by a continuous function $\bar{\eta}_i(x,t)$; i.e.,

$$|\eta_i(x,t)| \leq \bar{\eta}_i(x,t) \tag{4}$$

3 Fault Diagnosis

The present section discusses the fault diagnosis algorithm. First, an estimation model is designed. Subsequently, based on this model, a threshold bound is developed in order to generate a warning signal.

From (2), a nonlinear estimation model is built as,

$$\hat{x}^{(n)} = \Lambda \tilde{x}^{(n-1)} + F(x,t) + G(x,t)u, \tag{5}$$

where $\hat{x}^{(n)}$ represents the estimated state,

$\tilde{x}^{(n-1)} = x^{(n-1)} - \hat{x}^{(n-1)}$

represents the error state, and

$\Lambda = diag\{\lambda_1, \lambda_2, \ldots, \lambda_m\}\ (\lambda_i > 0)$

represents the gain matrix. Next, a residual signal is constructed. Utilizing (2) and (5), the following error dynamics can be developed:

$$\tilde{x}^{(n)} = -\Lambda \tilde{x}^{(n-1)} + \eta(x,t) + B(t-T)\zeta(x). \tag{6}$$

For convenience, the notation $\bar{x}_i = \tilde{x}_i^{(n_i-1)}$ is introduced. It follows that

$$x = -\Lambda x + \eta(x,t) + B(t-T)\zeta(x). \tag{7}$$

where $x = [\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m]^T$. According to equation (3), no fault occurs when $t < T$. This implies that

$$B(t-T)\zeta(x) = 0,\ \ time\ \ t < T. \tag{8}$$

Thus, each element $\bar{x}_i(t)$ of the residual vector follows that

$$\bar{x}_i(t) = e^{-\lambda_i t}\bar{x}_i(0) + \int_0^t e^{-\lambda_i(t-\tau)}\eta_i(x,t)d\tau,\ \ t < T.$$

Thus, an upper bound is obtained for each $\tilde{x}_i^{(n_i)}$ during $[0, T)$, that is,

$$\varpi_i = e^{-\lambda_i t}|\bar{x}_i(0)| + \int_0^t e^{-\lambda_i(t-\tau)}\bar{\eta}(x,t)d\tau. \tag{9}$$

Now, the following decision rule results:

When at least one element of the residual $|\bar{x}_i(t)|$ *goes beyond the threshold value* ϖ_i, *a fault has occurred.*

The fault detection time can be denoted as,

$$T_d = inf \cup_{i=1}^{m} \{t \,\|\, \bar{x}_i(t)| \geq \varpi_i\}. \tag{10}$$

A theoretical analysis is necessary to ensure that all faults are observable. The following theorem addressese the detectability of the fault detection, by characterizing the faults that can be detected, and shows that the fault will be detected before it goes to infinity (i.e., the system becomes unstable).

Theorem 3.1 (Detectability Issue): Suppose that there exists a time interval $[T_1, T_2](T \le T_1 < T_2)$ where $|\bar{x}_i(T_1)| \le \varpi_i$ and a scalar $M_i > 0$ (the range of M_i will be shown) such that at least one element of the fault term $B(t-T)\zeta(x)$ satisfies the following condition:

$$|\beta_i(t-T)\zeta_i(x)| > M_i + 2\bar{\eta}_i(x,t), \quad (11)$$

Then, a fault is detected; i.e., $|\bar{x}_i(T_2)| > \varpi_i$.

Proof. Notice that in this case each component of the estimation error equations satisfies,

$$\dot{\bar{x}}_i(t) = -\lambda_i \bar{x}_i(t) + \eta_i(x,t) + \beta_i(t-T)\zeta_i(x). \quad (12)$$

For any $t > 0$ $(T_1 + t \le T_2)$, the solution of (12) is

$$\bar{x}_i(T_1+t) = e^{-\lambda_i t}\bar{x}_i(T_1) \int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}\eta(x,\tau)d\tau$$

$$\int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}\beta_i(\tau-T)\zeta_i(x)d\tau.$$

Using the triangle inequality, it is obtained,

$$|\bar{x}_i(T_1+t)| \ge \left|\int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}\right.$$

$$\left.\beta_i(\tau-T)\zeta_i(x)d\tau\right| - e^{-\lambda_i t}|\bar{x}_i(T_1)|$$

$$\int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}\bar{\eta}_i(x,\tau)d\tau$$

Since $\beta_i(t-T)\zeta_i(x)$ is uniformly continuous, there exists a time interval $[T_1, T_2]$ such that $\beta_i(t-T)\zeta_i(x)$ retains the same sign for $t \in [T_1, T_2]$. Hence, for every $t \in [T_1, T_2]$, we have

$$|\bar{x}_i(T_1+t)| \ge \int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}$$

$$|\beta_i(\tau-T)\zeta_i(x)|\,d\tau - e^{-\lambda_i t}|\bar{x}_i(T_1)| -$$

$$\int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}\bar{\eta}_i(x,\tau)d\tau \ge \int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}$$

$$|\beta_i(\tau-T)\zeta_i(x)|\,d\tau - |\bar{x}_i(T_1)| -$$

$$\int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}\bar{\eta}_i(x,\tau)d\tau.$$

Using (11), we have

$$|\bar{x}_i(T_1+t)| \ge \int_{T_1}^{T_1+t} e^{-\lambda i(T+t-\tau)}M_i d\tau -$$

$$|\bar{x}_i(T_1)| = \lambda_i^{-1}(1-e^{-t})M_i - |\bar{x}_i(T_1)|. \quad (13)$$

This implies that if $M_i > 2\lambda_i [1 - e^{-(T_2 - T_1)}] - 1\ \varpi_i$ (note that $|\bar{x}_i(T_1)| \le \varpi_i$), then $|\bar{x}_i(T_2)| > \varpi_i$. This also implies that the fault is detected at the time $t = T_2$. Based on the above lemma, if a fault occurs, it will be detected at time T_d.

Remark 3.1. If he uncertain term $\eta_i(x,t)$ satisfying

$$|\eta_i(x,t)| \le k_1\|x\| + k_2 \quad (14)$$

is known, the threshold will have

$$\varpi_i = e^{-\lambda_i t}|\bar{x}_i(0)| + k_1\int_0^t e^{-\lambda_i(t-\tau)}\|x\|d\tau +$$

$$\frac{k_2}{\lambda_i}(1-e^{-\lambda_i t}). \quad (15)$$

If the uncertain term $\eta_i(x,t)$ satisfies the simple form,

$$|\eta_i(x,t)| \le k \quad (16)$$

the threshold will be given by

$$\varpi_i = e^{-\lambda_i t}|\bar{x}_i(0)| + \frac{k}{\lambda_i}(1-e^{-\lambda_i t}). \quad (17)$$

The detectability can be further analyzed by using a similar procedure as in *Theorem* 3.1.

4 Fault Tolerant Control

This section designs and analyzes the developed fault-tolerant controller. First, the original controller of the system (1) without fault is presented and the stability is discussed. Second, when a fault is present, a corrective control signal is added to the original controller, to achieve fault-tolerant control. Third, the closed-loop stability issues of the three different states are discussed.

4.1 Original control before fault detection

The objective of controller design is to achieve tracking control; i.e., following a desired reference signal $y_{di}(t) \in R$. The error $e_i(t)$ is expressed as $e_i = y_i - y_{di}$. Furthermore, the following filtered errors are designed:

$$\dot{s}_1 = \left(\frac{d}{dt} + k_1\right)^{n_1-1}e_1,$$

$$\dot{s}_2 = \left(\frac{d}{dt} + k_2\right)^{n_2-1}e_2,$$

$$\vdots$$

$$\dot{s}_m = \left(\frac{d}{dt} + k_m\right)^{n_m-1}e_m,$$

where $k_1, \dots, k_m$ are the designed filter gains.

According to the result of Slotine and Li [24], if $s_i(t) = 0$, this implies that their states e_i will approach 0 asymptomatically. Therefore, it is reasonable to use a filter error to represent the actual tracking error. Thus, the filtered system is given by,

$$S(t) = F(x,t) + G(x,t)u + v + \eta(x,t) + B(t-T)\zeta(x), \tag{18}$$

where,

$$S(t) = [s_1(t), s_2(t), s_3(t), \dots, s_m(t)]^T$$

$$v = [v_1, v_2, v_3, \dots, v_m]^T$$

$$v_i = -y_{di}^{(n_i)} + k_i^{n_i-1}\dot{e}_i + (n_i - 1)k_i^{n_i-2}\ddot{e}_i + \dots + (n_i - 1)k_i e_i^{(n_i-1)}.$$

When there is no fault, equation (18) is given by

$$S(t) = F(x,t) + G(x,t)u + v + \eta(x,t). \tag{19}$$

It is suggested to use the following control action:

$$u = G^{-1}(x,t)[-F(x,t) - v - \Lambda S - \frac{1}{2}\xi \| \bar{\eta}(x,t) \|^2 S], \tag{20}$$

where, Λ is the same as in (5) and ξ is a positive constant. The application of (20) to (19) produces,

$$S(t) = -\Lambda S - \frac{1}{2}\xi \| \bar{\eta}(x,t) \|^2 S + \eta(x,t). \tag{21}$$

Define the Lyapunov function $V_1 = S^T S$. V along (21) is given by,

$$V_1 = -2S^T\Lambda S - \xi \mathrm{P}\bar{\eta}(x,t)\mathrm{P}^2\mathrm{P}S\mathrm{P}^2 + 2S^T\eta(x,t). \tag{22}$$

Note that $2ab \le \xi a^2 + \xi^{-1}b^2$. Thus, we have

$$2S^T\eta(x,t) \le 2\mathrm{P}S\mathrm{P}\mathrm{P}\eta(x,t)P \le \xi\mathrm{P}\bar{\eta}(x,t)\mathrm{P}^2\mathrm{P}S\mathrm{P}^2 + \xi^{-1}.$$

Utilizing the above inequalities, (22) becomes as

$$V_1 \le -2\lambda_{min}(\Lambda)\| S \|^2 + \xi^{-1}, \tag{23}$$

where, $\lambda_{min}(\Lambda)$ is the smallest eigenvalue of Λ. Thus, whenever $\|S\| > \sqrt{\frac{\xi^{-1}}{2\lambda_{min}(\Lambda)}}$, V becomes negative. This demonstrates that the tracking errors S are uniformly ultimately bounded (UUB). Furthermore, from (23), it follows that,

$$V_1(t) \le \frac{\xi^{-1}}{2\lambda_{min}(\Lambda)} + [V_1(0) - \frac{\xi^{-1}}{2\lambda_{min}(\Lambda)}]e^{-2\lambda_{min}(\Lambda)t} \tag{24}$$

Thus, it follows that

$$\|S\| \le \sqrt{\frac{\xi^{-1}}{2\lambda_{min}(\Lambda)} + [V_1(0) - \frac{\xi^{-1}}{2\lambda_{min}(\Lambda)}]e^{-2\lambda_{min}(\Lambda)t}},$$

$$\lim_{t\to\infty}\|S\| \le \sqrt{\frac{\xi^{-1}}{2\lambda_{min}(\Lambda)}} \tag{25}$$

Therefore, we have the following stability result.

Theorem 4.1. (Stability Issue In the Absence of Fault) Suppose that the assumptions A1- A3 hold. Apply the controller (20) to the system (1) without fault occurrence. Then, the tracking errors S are UUB, and S satisfy the property (25).

Remark 4.1. The guidelines for choosing the parameters in the theorem are given now. Increasing ξ can help reduce the bound of $\|S\|$. However, if ξ is too large, it will lead to high-gain control, which is not desirable in a closed-loop system. Therefore, in a practical application, a trade-off has to be made for achieving suitable transient performance and control action.

Suppose that a fault has occurred, but it has not been detected. In this case, the fault may lead to system instability since the corrective control is not activated. To solve this problem, we will now consider the system stability if a fault has occurred but not detected under the original control scheme.

Theorem 4.2 (Stability Issue Before Fault Detection): Assume that a fault occurs at T. Then, for $t \in [T, T_d)$, the robust controller (20) can ensure that the tracking error s_i is bounded.

Proof. For $T + t \in [T, T_d)$, the closed-loop system becomes,

$$S(t) = -\Lambda S - \frac{1}{2}\xi \| \bar{\eta}(X,t) \|^2 S + \eta(x,t) + B(t-T)\zeta(x).$$

Each component of the closed-loop system is given by

$$\dot{s}_i = -\lambda_i s_i + \eta_i(x,t) - \frac{1}{2}\xi \| \bar{\eta}(x,t) \|^2 s_i + \beta_i(t-T)\zeta_i(x).$$

Thus, we have

$$s_i(T+t) = e^{-\lambda_i t} s_i(T) + \int_T^{T+t} e^{-\lambda i(T+t-\tau)} [\eta_i(x,\tau) - \frac{1}{2}\xi \| \bar{\eta}(x,\tau) \| s_i] d\tau + \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \beta_i(\tau - T)\zeta_i(x) d\tau.$$

It follows that

$$| s_i(T+t) | \le e^{-\lambda_i t} | s_i(T) | + \int_T^{T+t} e^{-\lambda i(T+t-\tau)} [\bar{\eta}_i + \frac{1}{2}\xi \| \bar{\eta}(x,\tau) \|^2 | s_i |] d\tau + | \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \beta_i(\tau - T)\zeta_i(x) d\tau | \le e^{-\lambda_i t} | s_i(T) | + \sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \int_T^{T+t} e^{-\lambda i(T+t-\tau)} d\tau + \frac{1}{2}\xi \sup_{\tau \in [T,T_d]} \{ \| \bar{\eta}(\tau) \|^2 \} \int_T^{T+t} e^{-\lambda i(T+t-\tau)} | s_i | d\tau + | \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \beta_i(\tau - T)\zeta_i(x) d\tau | \le e^{-\lambda_i t} | s_i(T) | + \sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \lambda_i^{-1} + \frac{1}{2}\xi \sup_{\tau \in [T,T_d]} \{ \| \bar{\eta}(\tau) \|^2 \} \int_T^{T+t} e^{-\lambda i(T+t-\tau)} | s_i | d\tau + | \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \beta_i(\tau - T)\zeta_i(x) d\tau |. \quad (26)$$

where, Assumption A3) has been used. Recalling (12), the solution of $\bar{x}_i$ for time $t \in [T, T_d)$ is,

$$\bar{x}_i(T+t) = e^{-\lambda_i t}\bar{x}_i(T) + \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \eta_i(x,t) d\tau + \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \beta_i(\tau - T)\zeta_i(x,\tau) d\tau.$$

The triangle inequality follows that,

$$| \bar{x}_i(T+t) | \ge | \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \beta_i(\tau - T)\zeta_i(x,\tau) d\tau | - e^{-\lambda_i t} | \bar{x}_i(T) | - \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \bar{\eta}_i(x,t) d\tau \ge | \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \beta_i(\tau - T)\zeta_i(x,\tau) d\tau | - | \bar{x}_i(T) | - \sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \lambda_i^{-1}. \quad (27)$$

Since for $t \in [T, T_d)$ the fault has not been detected, this implies that $| \bar{x}_i(T+t) | \le \varpi_i$ and $| \bar{x}_i(T) | \le \varpi_i$. It follows from (27) that,

$$| \int_T^{T+t} e^{-\lambda i(T+t-\tau)} \beta_i(\tau - T) \zeta_i(x,\tau) d\tau | \le 2\varpi_i + \sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \lambda_i^{-1}.$$

Substitute the above inequality into (26), to get

$$| s_i(T+t) | \le e^{-\lambda_i t} | s_i(T) | + 2 \sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \lambda_i^{-1} + 2\varpi_i + \frac{1}{2}\xi \sup_{\tau \in [T,T_d]} \{ \| \bar{\eta}(\tau) \|^2 \} \int_T^{T+t} e^{-\lambda i(T+t-\tau)} | s_i | d\tau.$$

According to B_G lemma [23], it follows that

$$| s_i(T+t) | \le [| s_i(T) | + 2 \sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \lambda_i^{-1} + 2\varpi_i] e^{-\lambda_i t} e^{\frac{1}{2}\xi \sup_{\tau \in [T,T_d]} \{ \| \bar{\eta}(\tau) \|^2 \} t} + 2\lambda_i [\sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \lambda_i^{-1} + \varpi_i] \times \int_T^{T+t} e^{-\lambda i(T+t-\tau)} e^{\frac{1}{2}\xi \sup_{\tau \in [T,T_d]} \{ \| \bar{\eta}(\tau) \|^2 \} (T+t-\tau)} d\tau = [| s_i(T) | + 2 \sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \lambda_i^{-1} + 2\varpi_i] \times e^{-(\lambda_i - \frac{1}{2}\xi \sup_{\tau \in [T,T_d]} \{ \| \bar{\eta}(\tau) \|^2 \}) t} + \frac{2\lambda_i (\sup_{\tau \in [T,T_d]} \{ \bar{\eta}_i(\tau) \} \lambda_i^{-1} + \varpi_i)}{(\lambda_i - \frac{1}{2}\xi \sup_{\tau \in [T,T_d]} \{ \| \bar{\eta}(\tau) \|^2 \})} \times [1 - e^{-(\lambda_i - \frac{1}{2}\xi \sup_{\tau \in [T,T_d]} \{ \| \bar{\eta}(\tau) \|^2 \}) t}]. \quad (28)$$

Note that $t \in [0, T_d - T)$. Thus, $s_i(T+t)$ is bounded for $T + t \in [T, T_d)$. This bound can be reduced by increasing the value λ_i .

4.2 Controller reconfiguration: fault-tolerant control after fault is detected

Once a fault is detected, it is necessary to further reconfigure the controller so that the fault effects can be reduced significantly. How to reconfigure the controller and the the stability issues when using the proposed controller are addressed now.

After the occurrence of a fault, the system equation (19) becomes,

$$S(t) = F(x,t) + G(x,t)u + v + \eta(x,t) + B(t - T)\zeta(x), \tag{29}$$

where,

$$B(t - T) = I - \Theta(t - T), \ t \geq T, \tag{30}$$

with $\Theta(t - T) = diag\{e^{-\theta_1(t-T)}, e^{-\theta_2(t-T)}, \dots, e^{-\theta_n(t-T)}\}$.

If $\zeta(x)$ is not available, it implies that $B(t - T)\zeta$ is also unknown. In this situation, a linearly parameterized approximator is suggested to approximate the unknown function $\zeta(x)$. Several function approximators can be applied for this purpose; for example, fuzzy logic systems, polynomials, and neural networks, which can be represented as $W^T\Phi(z)$ with input vector z , weight vector W , node number l , and basis function vector $\Phi(z)$. The universal approximation theorem indicates that, if l is chosen sufficiently large, then $W^T\Phi(z)$ can approximate any continuous function to any desired accuracy over a compact set [19]. Thus, $\zeta(x)$ can be expressed approximately by a neural network (NN) [20, 21, 22]

$$\zeta(x) = W^{*T}\Phi(x) + \varepsilon, \tag{31}$$

where ε represents the function approximation error, satisfying $\|\varepsilon\| \leq \varepsilon_M$ with constant ε_M , and the ideal weight W^* can be obtained by

$$W^* := argmin_{W \in \Omega_W}\{\sup_{x \in \Omega_q} \| W^T\Phi(x) - \zeta(x) \|\}. \tag{32}$$

Unfortunately, ε is unknown and it is not possible to obtain the value of W^* . For this situation, an adaptive controller is developed to cope with the unknown weight.

Denote $\hat{W}$ and $\hat{\zeta}(x)$ as the estimates of W^* and $\zeta(x)$, respectively. Thus, we have

$$\hat{\zeta}(x) = \hat{W}^T\Phi(x). \tag{33}$$

Therefore, the reconfigured controller is

$$u = G^{-1}(x,t)[-F(X,t) - v - \Lambda S - \frac{1}{2}\xi \| \bar{\eta}(X,t) \|^2 S - \hat{W}^T\Phi(x)], \tag{34}$$

with the neural network learning mechanism,

$$\dot{\hat{W}} = \Upsilon\Phi(x)S^T - \rho\Upsilon(\hat{W} - W_a), \tag{35}$$

where, Υ, W_a , and $\rho >$ are design parameters. The closed-loop system subject to the controller (34) becomes,

$$S(t) = -\Lambda S - \frac{1}{2}\xi \| \bar{\eta}(X,t) \|^2 S + \eta(x,t) + B(t - T)\zeta(x) - \hat{W}^T\Phi(x)$$

It should be noticed that $B(t - T)\zeta(x) - \hat{W}^T\Phi(x)$ can be further derived:

$$B(t - T)\zeta(X) - \hat{W}^T\Phi(x) = B(t - T)W^{*T}\Phi(x) - \hat{W}^T\Phi(x) + B(t - T)\varepsilon = \tilde{W}^T\Phi(x) - \Theta(t - T)W^{*T}\Phi(x) + B(t - T)\varepsilon, \tag{36}$$

where $\tilde{W} = W^* - \hat{W}$. Clearly, each element of $B(t - T)$ is a bounded time function. This implies that $\| B(t - T)\varepsilon \| \leq \varepsilon_M$. It follows that,

$$S(t) = -\Lambda S - \frac{1}{2}\xi \| \bar{\eta}(X,t) \|^2 S + \eta(x,t) + \tilde{W}^T\Phi(x) - B(t - T)\varepsilon + \Theta(t - T)W^{*T}\Phi(x). \tag{37}$$

For the stability analysis, a Lyapunov function is designed. Denote a candidate as $V = V_1 + tr(\tilde{W}^T\Upsilon^{-1}\tilde{W})$. Using (23), the time derivative of V is given by,

$$V \leq -2\lambda_{min}(\Lambda) \| S \|^2 + \xi^{-1} + 2S^T\tilde{W}^T\Phi(x) - 2S^TB(t - T)\xi + 2S^T\Theta(t - T)W^{*T}\Phi(x) - 2tr[\tilde{W}^T\Upsilon^{-1}\hat{W}] = -2\lambda_{min}(\Lambda) \| S \|^2 + \xi^{-1} + 2tr[\tilde{W}^T\Phi(x)S^T - \tilde{W}^T\Upsilon^{-1}\hat{W}] - 2S^TB(t - T)\varepsilon + 2S\Theta(t - T)W^{*T}\Phi(x) = -2\lambda_{min}(\Lambda) \| S \|^2 + \xi^{-1} + 2\rho tr[\tilde{W}^T(\hat{W} - W_a)] -$$

$2S^TB(t-T)\varepsilon + 2S^T\Theta(t-T)W^{*T}\Phi(x)$. (38)

By completing the squares, we have

$2tr[\tilde{W}^T(\hat{W}-W_a)] =$

$2tr[\tilde{W}^T(W^*-W_a-\tilde{W})] =$

$-2\|\tilde{W}\|_F^2 + 2tr[\tilde{W}^T(W^*-W_a)] \le$

$-2\|\tilde{W}\|_F^2 + \|\tilde{W}\|_F^2 + \|W^*-W_a\|_F^2 =$

$-\|\tilde{W}\|_F^2 + \|W^*-W_a\|_F^2$.

where, the symbol $\|\cdot\|_F$ denotes the Frobenius norm. Utilizing the formula $2\alpha^T\beta \le \frac{1}{2}\alpha^T\alpha + 2\beta^T\beta$, we have

$-2S^TB(t-T)\xi \le \frac{1}{2}\lambda_{min}(\Lambda)\|S\|^2 +$

$2\lambda_{min}^{-1}(\Lambda)|B(t-T)\varepsilon|^2 \le$

$\frac{1}{2}\lambda_{min}(\Lambda)\|S\|^2 + 2\lambda_{min}^{-1}(\Lambda)\varepsilon_M^2$

$2S^T\Theta(t-T)W^{*T}\Phi(x) \le \frac{1}{2}\lambda_{min}(\Lambda)\|S\|^2 +$

$2\lambda_{min}^{-1}(\Lambda)\|\Theta(t-T)W^{*T}\Phi(x)\|^2 \le$

$\frac{1}{2}\lambda_{min}(\Lambda)\|S\|^2 +$

$2\lambda_{min}^{-1}(\Lambda)max_{1\le i\le n}[e^{-2\theta_i(t-T)}]\|W^{*T}\Phi(x)\|^2$.

Applying the above equations to (38), it follows that,

$V \le -\lambda_{min}(\Lambda)\|S\|^2 - \rho\|\tilde{W}\|_F^2 + \rho\|W^* -$
$W_a\|_F^2 + 2\lambda_{min}^{-1}(\Lambda)\varepsilon_M^2 + 2\lambda_{min}^{-1}(\Lambda)max_{1\le i\le n}$
$[e^{-2\theta_i(t-T)}]\|W^{*T}\Phi(q,\dot{q})\|^2 + \xi^{-1}$.

Let,

$$\mu = 2\lambda_{min}^{-1}(\Lambda)\varepsilon_M^2 + 2\lambda_{min}^{-1}(\Lambda)\times max_{1\le i\le n}[e^{-2\theta_i(t-T)}]\|W^{*T}\Phi(x)\|^2 + \xi^{-1}. \quad (39)$$

Thus, we have $V \le 0$ if

$$\|S\| > \sqrt{\frac{\rho\|W^*-W_a\|_F^2+\mu}{\lambda_{min}(\Lambda)}},$$

$$or, \ \|\tilde{W}\|_F > \sqrt{\frac{\rho\|W^*-W_a\|_F^2+\mu}{\rho}}.$$

This implies that $S, \tilde{W}$ are uniformly bounded. Moreover, define $\bar{\lambda} = min\{\lambda_{min}(\Lambda), \rho\lambda_{min}(\Upsilon)\}$. Since,

$$V \le \|S\|^2 + \frac{1}{\lambda_{min}(\Upsilon)}\|\tilde{W}\|_F^2,$$

we have,

$$V \le -\bar{\lambda}V + \rho\|W^*-W_a\|_F^2 + \mu.$$

Furthermore,

$$\|S\| \le \sqrt{\frac{\rho\|W^*-W_a\|_F^2+\mu}{} + \Omega e^{-\bar{\lambda}(t-T)}}, \quad t \ge T_d. \quad (40)$$

where,

$$\Omega = V(0) - \frac{\rho\|W^*-W_a\|_F^2+\mu}{\bar{\lambda}} \quad (41)$$

Therefore, the following theorem is established.

Theorem 4.3 (System Stability After Fault Detection). Suppose that the Assumptions A1- A3 hold. Apply the controller (34) with (35) to the system (1), when considering the fault occurrence. Then, both the state error vector S and the weight vector $\tilde{W}$ are uniformly ultimately bounded, and S satisfies the property (40).

Remark 3.2. In this section, the NN approximator is designed to cope with the fault occurrence. Since NNs have learning capabilities, the proposed fault-tolerant control can ensure that the closed-loop system is stable (see Theorem 4.3). One way to improve the fault-tolerant control performance is to reduce the NN approximation error ξ. This can be achieved by increasing the number of nodes in the NN. The bound (39) implies that decreasing the approximation error can reduce the value of μ. This also reduces the error S.

In order to implement the fault-tolerant controller (34), the matrix $G(x)$ must be invertible, as described in A2), and in turn the developed controller can be well defined. Now, a modified fault accommodation controller without requiring the assumption A2) wis constructed. Here, the following assumptions are made.

A4) $G(x,t)$ is positive definite or negative definite.

This condition guarantees that the nonlinear system (2) is strong controllable. Following a similar procedure of proof as in Theorem 4.1 or Theorem 4.

3, the following stability theorems are established for the modified fault accommodation control scheme.

Theorem 4.4 (Stability Issue Without Fault) Suppose that the assumptions A1, A3, A4 hold. Apply the following controller to system (1) without considering faults and the original control:

$$u = \frac{S}{S^T G(x,t) S}[-k_\Lambda \| S \|^2 - S^T F(x,t) - S^T v - \frac{1}{2}\xi \| \bar{\eta}(x) \|^2 \| S \|^2]$$

Then, the tracking errors S are UUB, and S satisfies the property

$$lim_{t\to\infty}\|S\| \le \sqrt{\frac{\xi^{-1}}{2k_\Lambda}}.$$

Theorem 4.5 (Stability Issue Before Fault Detection): Assume that a fault occurs at T but not detected. Suppose that the assumptions A1, A3, A4 hold. Apply the following controller to system (1) and the original control signal:

$$u = \frac{S}{S^T G(x,t) S}[-k_\Lambda \| S \|^2 - S^T F(x,t) - S^T v - \frac{1}{2}\xi \| \bar{\eta}(x) \|^2 \| S \|^2]$$

Then, for $t \in [T, T_d)$, the above controller can ensure that the tracking error s_i is bounded.

Theorem 4.6 (Stability Issue After Fault Detection). Suppose that the assumptions A1, A3, A4 hold. Apply the following fault-tolerant controller to the system (1) with considering faults:

$$u = \frac{S}{S^T G(x,t) S}[-k_\Lambda \| S \|^2 - S^T F(x,t) - S^T v - \frac{1}{2}\xi \| \bar{\eta}(x) \|^2 \| S \|^2 - S^T \hat{W}^T \Phi(x)]$$

Then, both the error vector S and the weight vector $\tilde{W}$ are uniformly ultimately bounded.

5 Case Study

This section presents an example to illustrate the performance of the fault diagnosis and fault-tolerant control scheme that has been developed in the present work.

Consider the following system:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.1\dot{x}_1 - 0.1\dot{x}_2 \\ -0.1\dot{x}_2 - 0.1\dot{x}_1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \eta(x_1,\dot{x}_1,x_2,\dot{x}_2) + B(t-T)\begin{bmatrix} \zeta_1(x_1,\dot{x}_1,x_2,\dot{x}_2) \\ \zeta_2(x_1,\dot{x}_1,x_2,\dot{x}_2) \end{bmatrix},$$

$$y_1 = x_1,\ y_2 = x_2$$

where, $\eta_1 = 0.5cos(x_1)cos(x_2)$, $\eta_2 = 0.5sin(x_2)sin(x_1)$ which are assumed to be unknown and bounded by $|\eta_1| \le 1 = \bar{\eta}_1$, $|\eta_2| \le 1 = \bar{\eta}_2$. First, use the fault estimator proposed in (5), and choose the gain Λ as $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. The threshold value is computed using,

$$\varpi_1 = \varpi_2 = e^{-4t} + \frac{1}{4}[1 - e^{-4t}]$$

when $|\tilde{x}_i(0)| \le 1$, $i = 1,2$. During the test, the first fault function is described by,

$$\zeta_1(x_1,\dot{x}_1,x_2,\dot{x}_2) = [1 - e^{-10(t-T)}] \times (3x_1^2\dot{x}_1x_2^2 + 10),$$

while the second fault function is zero. The fault occurs at $T = 5s$. The original controller is designed according to (20), where $k_1 = k_2 = 10, \xi = 1$, and the desired trajectories are $y_{d1} = sin(\pi t)$ and $y_{d2} = cos(\pi t)$ for y_1 and y_2, respectively. Fig. 2 shows the control results and fault histories without considering fault-tolerant scheme. From this figure, it is observed that the errors of $y_1 - y_{1d}$ increase significantly after the fault occurrence. It should be noticed that the residual has exceeded the threshold, and the fault has been detected at $T_d = 5.1564$. Now we trigger the proposed fault-tolerant controller after the fault is detected. The parameters Υ and ρ in the neural network learning are first fixed at $0.5I$ and $\rho = 0.02$, respectively. The total number of NN nodes is $l = 80$. The NN basis is chosen as $\Phi = [\varphi_1, \varphi_2, \ldots, \varphi_l]^T$ with $\varphi_i = exp$

$$(-\frac{\| [x_1,\dot{x}_1,x_2,x\dot{x}_2]^T - [c_{i1},c_{i2},c_{i3},c_{i4}]^T \|^2}{2\sigma_i^2}).$$

Therefore, the neural network function contains

l nodes whose centers are at c_{ij} ($i = 1,\dots,l; j = 1,2,3,4.$) evenly spaced in [-1,1], [-5,5], [-1,1], [-5,5], respectively, and widths $\sigma_i = 5$. The initial neural network weight vector is selected as $\hat{W}(0) = W_a = 0.0$. Fig. 3 presents the performance of the fault-tolerant controller. It is apparent that the state tracking performance is satisfactory. This verifies that the NN learning can reduce the negative effect of the fault occurrence.

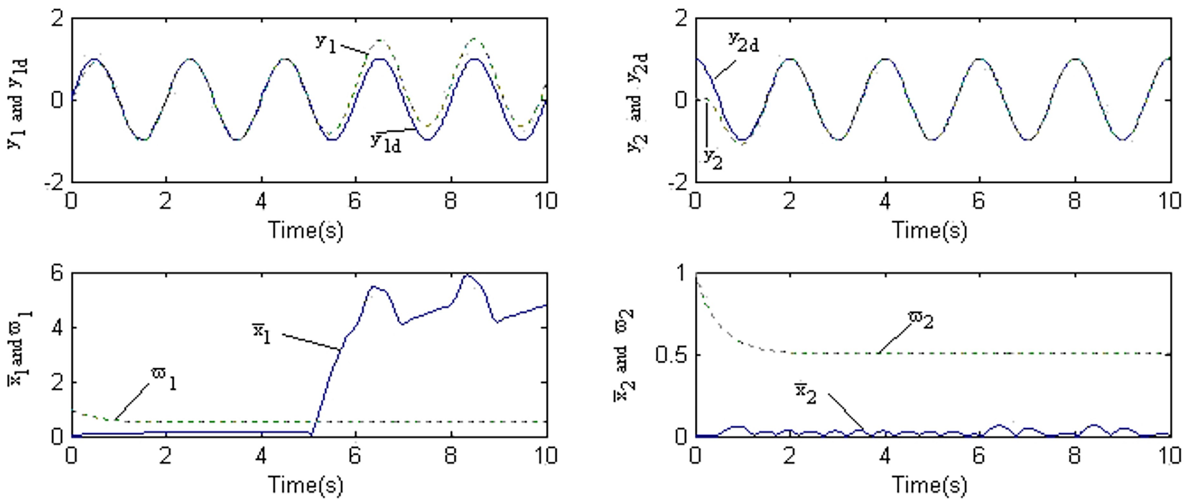

Fig. 2 Simulation results: original control without fault-tolerance (top); fault diagnosis (bottom)

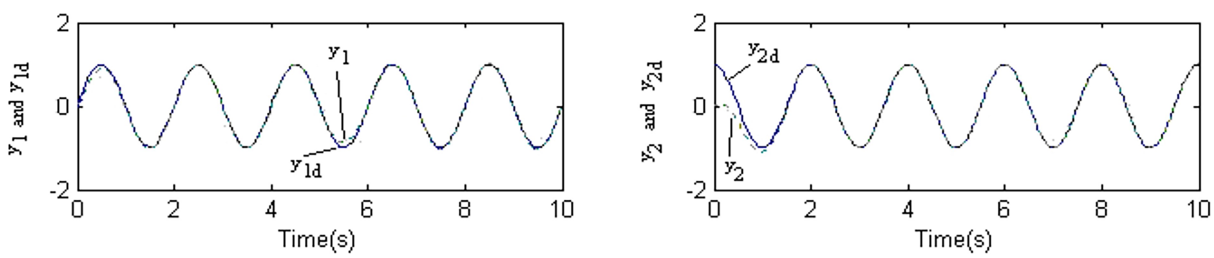

Fig. 3 Simulation results: fault-tolerant control

6 Conclusions

A fault detection and fault-tolerant control method was developed and presented in this paper. Using an observer model, the monitoring system could send a reminder signal when a fault was detected. After receiving the signal, the NN-based fault-tolerant control was triggered to continue maintaining proper operation and minimizing the effect of the fault. Future research will carry out an experimental test to further verify the performance of the proposed approach.

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