Bistable Piezoelectric Flutter Energy Harvesting with Uncertainty

Farbod KHOSHNOUD[1], Christopher R. BOWEN[2], Cristinel MARES[3]

(1.*Department of Electromechanical Engineering Technology*, *College of Engineering*, *California State Polytechnic University*, *Pomona*, *CA* 91768, *USA*;

2.*Department of Mechanical Engineering*, *University of Bath*, *Bath*, *BA*2 7*AY*, *UK*;

3.*Department of Mechanical and Aerospace Engineering*, *Brunel University London*, *Uxbridge*, *UB*8 3*PH*, *UK*)

Abstract: The analytical formulation of piezoelectric flutter energy harvesting using a bistable material, while considering uncertainties in the model is presented in this paper. Bistable laminates provide the advantage of large deflection due to the nonlinear snap-through characteristics when exposed to external loading, and can therefore provide a suitable base for piezoelectric material in energy harvesting applications. A piezoelectric material that is bounded on the surface of bistable laminates, subjected to external loading, generates large strains and hence relatively higher electrical output energy, in comparison with the case where piezoelectric material is bonded on a regular surface, with analogous loading conditions. Although information regarding the external loading, material characteristics of the bistable laminate and the piezoelectric material, boundary conditions, and overall electrical circuit efficiency can be defined for analytical purposes, the exact model of the system is not readily accessible. The unavoidable uncertainties in the material, loading, and efficiency of a complex system call for a probabilistic approach. Hence, this paper provides a formulation that considers uncertainty bounds in obtaining a realistic model. Optimal Uncertainty Quantification (OUQ) is used in this paper, which takes into account uncertainty measures with optimal bounds and incomplete information about the system, as a well-defined optimization problem according to maximum probabilities, subjected to the imposed constraints. The OUQ allows the inspection of the solution for a span of uncertain input parameters, as a reliable and realistic model.

Key words: Bistable laminates, piezoelectric energy harvesting, flutter energy harvesting, optimal uncertainty quantification, self-powered systems.

1 Introduction

Energy independent systems are realized by implementing energy harvesting techchniques, in the development of self-powered sensors and actuators[1]-[4], using electromagnetic[5]-[9], piezoelectric[10]-[12], or electrostatic[10] energy conversion mechanisms. This is a power supply solution particularly for low-powered electronics, (e.g. with application in wireless sensor networks and monitoring devices), and renewable energy sources[1]-[4], [13].

Various techniques have been employed by researchers to make use of such piezoelectric energy harvesting systems with optimum output results. There is an extensive literature on energy harvesting, and some examples are indicated in the following brief review.

Techniques that allow improved outputefficiency are included in multiple-frequency energy harvesting across a wide band[14]-[15]. For example, external magnets are used to provide stiffness nonlinearities[16], which tune the operating frequencies, and broaden the frequency bandwidth. Also, temperature-dependent bistable laminates are used[17], with significant power outputs over a wide bandwidth by utilizing square bistable laminates[18]-[21].

Static stable models are obtained by identifying the coefficients of assumed polynomials and minimizing the total strain energy. This has been validated experimentally and also using finite element analysis for square and rectangular laminates[21]-[23]. The double-well potential energy dynamic model of the system[24] predicts the outputs for any given vibration pattern. Such modeling has been applied to bistable laminates of trapezoidal and triangular planforms [25] for morphing applications, and have been modelled analytically as tapered cantilever configurations [26], and in arbitrary shapes [27]-[28].

The unavoidable uncertainties in the material, loading, and efficiency of a complexbistable piezoelectric flutter energy harvester call for a probabilistic approach. Hence, this paper offers a formulation that considers uncertainty bounds in obtaining a realistic model. Optimal Uncertainty Quantification (OUQ)[29] is used in this paper, which takes into account uncertainty measures with optimal bounds and incomplete information about the system, as a well-defined optimization problem according to probability maximization, subjected to the imposed constraints. In this paper, an analytical model of nonlinear piezoelectric bistable laminate is introduced. Next, the dynamic model of the system subjected to aerodynamic loading is presented. Finally, the stochastic model of the system with bounded constraints is formulated, which can inspect the solution for a span of uncertain input parameters, as a reliable and realistic model.

2 Nonlinear Dynamics of Piezoelectric Bistable Laminates

An analytical, geometrically nonlinear model for piezoelectric bistable laminates with nonlinear electromechanical coupling [27]-[28] is presented in this section.

2.1 Constitutive equations for piezoelectric plates

The assumptions made in modeling the nonlinear piezoelectric bistable laminates are as follows[27]-[28]:

1 Bistable laminates are considered as platestructures.

2 Only the laminate is cured at high temperature and then the piezoelectric layer is attached.

3 The piezoelectric material is polarized along its thickness.

4 The Kirchhoff plate theory is applicable.

The constitutive equations for piezoelectric material may be expressed as

$$\begin{cases} \sigma = C\varepsilon - eE \\ D = e\varepsilon + \eta E \end{cases} \tag{1}$$

where σ and ε are stress and strain fields, respectively, C is the elasticity matrix, e is the electromechanical coupling coefficients matrix, E is the electrical field vector, D is the electrical displacement vector, and η is the premittivity components matrix. Based on the Kirchhoff theory, with $\sigma_{33} = 0$, for plate structures, the constitutive relation can be given by,

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ D_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{16} & e_{13} \\ C_{21} & C_{22} & C_{26} & e_{23} \\ C_{61} & C_{62} & C_{66} & 0 \\ e_{31} & e_{32} & 0 & -\eta_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \\ -E_3 \end{bmatrix} \tag{2}$$

The shear forces perpendicular to the mid plane are assumed to be negligible. Axial forces (N) and bending moments (M) are given by the expressions

$$\begin{cases} N = (N_{11} \quad N_{22} \quad N_{12})^{\mathrm{T}} = \int_{t_1^p}^{t_2^p} \sigma^0 \mathrm{d}z \\ M = (M_{11} \quad M_{22} \quad M_{12})^{\mathrm{T}} = \int_{t_1^p}^{t_2^p} z\sigma \mathrm{d}z \end{cases} \tag{3}$$

where $t_2^p - t_1^p$ is the thickness of the piezoelectric layer and σ^0 and σ are the membrane and bending parts, respectively, of the section stress, which is given by,

$$\sigma^0 = C\,\varepsilon^0 \; ; \; \sigma = zC\kappa \tag{4}$$

where ε^0 and κ are in-plane strains and curvatures.

By substituting (2) into (3), the constitutive relations for a cross-ply laminate in a general and compact form may be expressed as,

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} - (t_2^p - t_1^p) \begin{bmatrix} I_3 \\ \dfrac{(t_2^p + t_1^p)}{2} I_3 \end{bmatrix} \begin{bmatrix} e_{31} \\ e_{32} \\ 0 \end{bmatrix} E_3 \tag{5}$$

where A, D and B denote the membrane (extensional), bending and coupling stiffness matrices, respectively, and I_3 is the identity matrix. By integrating both sides of the sensing equation over the thickness

$$(t_2^p - t_1^p) D_3 = [e_{31} \quad e_{32} \quad 0] (t_2^p - t_1^p) \begin{bmatrix} I_3 & \frac{(t_2^p + t_1^p)}{2} I_3 \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} + \varepsilon_{33} E_3 (t_2^p - t_1^p) \quad (6)$$

By cancelling the piezoelectric thickness from bothsides of Equation (6),

$$\begin{cases} \bar{N} = Q \bar{\varepsilon} - t^p e' E_3 \\ D_3 = e'^T \bar{\varepsilon} + \varepsilon_{33} E_3 \end{cases} \quad (7)$$

where $\bar{N} = \begin{bmatrix} N \\ M \end{bmatrix}$, $\bar{\varepsilon} = \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix}$, $Q = \begin{bmatrix} A & B \\ B & D \end{bmatrix}$ $e' = \begin{bmatrix} I_3 \\ \frac{\bar{t}}{2} I_3 \end{bmatrix} \begin{bmatrix} e_{31} \\ e_{32} \\ 0 \end{bmatrix}$, and $\bar{t} = t_2^p + t_1^p$, $t^p = t_2^p - t_1^p$.

Seperation of the linear and nonlinear parts of the strain field leads to

$$\bar{\varepsilon} = \bar{\varepsilon}^L + \bar{\varepsilon}^{NL};$$
$$\bar{\varepsilon}^L = \begin{bmatrix} \varepsilon^{0L} \\ \kappa \end{bmatrix}; \bar{\varepsilon}^{NL} = \begin{bmatrix} \varepsilon^{0NL} \\ 0 \end{bmatrix} \quad (8)$$

Rewriting(7) in terms of linear and nonlinear parts,

$$\begin{cases} \bar{N} = Q \bar{\varepsilon}^L + Q \bar{\varepsilon}^{NL} - t^p e' E_3 \\ D_3 = e'^T \bar{\varepsilon}^L + e'^T \bar{\varepsilon}^{NL} + \varepsilon_{33} E_3 \end{cases} \quad (9)$$

2.2 Constitutive equations for bistable laminate

The constitutive equation forbistable laminates may be expressed as follows

$$\bar{N}^s = Q^s \bar{\varepsilon}^s \quad (10)$$

wherethe superscript s stands for substructure. This equaiton in terms of linear and nonlinear parts of the relation, can be writen as

$$\bar{N}^s = Q^s \bar{\varepsilon}^{L,s} + Q^s \bar{\varepsilon}^{NL,s} \quad (11)$$

Thein-plane strain filed and out of plane displacement component (deflection) are approximated based on Hyer's model [1] as,

$$[\varepsilon_x^0 \quad \varepsilon_y^0 \quad \gamma_{xy}^0] \approx \bar{\varphi} d + f \approx \varepsilon^{0L} + \varepsilon^{0NL} \quad (12)$$

where $\bar{\varphi}$ is the approximation function matrix for linear strains and f is the nonlinear part of the approximation.

Using in-plane strain field approximation in (12), $\bar{\varepsilon}^L$ and $\bar{\varepsilon}^{NL}$, from Equation (8) may be expressed in terms of the elongation coefficients d and the curvature coefficients a in a matrix form as follows:

$$\bar{\varepsilon}^L = \begin{bmatrix} \varepsilon^{0L} \\ \kappa \end{bmatrix} \approx \begin{bmatrix} \bar{\varphi} & 0 \\ 0 & -I_3 \end{bmatrix} \begin{bmatrix} d \\ a \end{bmatrix} = \Phi X \quad (13)$$

$$\bar{\varepsilon}^{NL} = \begin{bmatrix} \varepsilon^{0NL} \\ 0 \end{bmatrix} \approx [00f000]^{\mathrm{T}} = \begin{bmatrix} f \\ 0 \end{bmatrix} = \Psi \quad (14)$$

Now, $\bar{\varepsilon}$ in the constitutive relations is substituted by its approximated values as,

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} = \bar{\varepsilon}^L + \bar{\varepsilon}^{NL} = \Phi X + \Psi \quad (15)$$

2.3 Derivation ofthe governing dynamic equations

To satisfy the dynamic equilibrium of thebistable laminate with bonded piezoelectric elements, according to Hamilton's principle, the potential energy of the structure under dynamic conditions is minimized as follows:

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W_{ie} + \delta W_{nc}) \, \mathrm{d}t = 0 \quad (16)$$

The energy terms in Equation(16) for the nonlinear piezoelectric composite plate can be identified. The total strain energy of the system may be written as,

$$U = U^s + U^p = U^{m,s} + U^{th,s} + U^{m,p} + U^{e,p} \quad (17)$$

where, superscripts s, p, m, th, and e stand for substructure, piezoelectric, mechanical, thermal and electrical strain energies, respectively.

3 PiezoelectricFlutter 2DOF

A simplified two-degree-of-freedom (2DOF) lumped parameter model of piezoelectric flutter energy harvesting is discussed now. It will be further expanded to a continuous model associated with the bistable laminate as a basis of the piezoeelctric part, in the next section. The degrees of freedom for the aeroelastic vibration of a piezoelectric energy harvester [31]-[33], when exposed to uniform incompressible potential flow, can be defined by the plunge displacement or translation h, and the pitch displacement or rotation α as shown in Fig. 1, which are defined using aircraft wing flutter analysis. Also, h and α are the displacements of a base plate that is responsible for the input excitation of the bounded piezoelectric material. The optimization problem is expressed as

$$\int_{t_2}^{t_1} (\delta T - \delta U + \delta W_{ie} + \delta W_{nce} + \delta W_{nca} + \delta W_{sd})\, dt = 0 \quad (18)$$

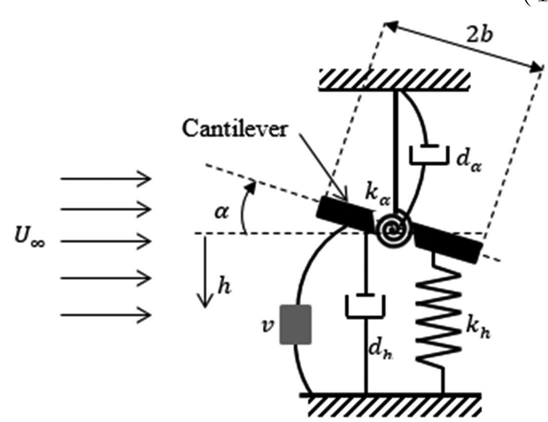

Fig. 1 A 2DOF model for aeroelastic vibration [2].

where T and U are the total kinetic and potential energies per unit length, W_{ie} denotes the internal electrical energy per unit length, W_{nce} is the virtual work component of the non-conservative electrical charge, W_{nca} is the virtual work component of the aerodynamic loads, and W_{sd} is the virtual work of the structural damping.

The kinetic energy of the system per length can bewritten as

$$T = \frac{1}{2} m(\dot{h}^2 + 2 x_\alpha b \dot{h} \dot{\alpha}) + \frac{1}{2} m_f \dot{h}^2 + \frac{1}{2} I_p \dot{\alpha}^2 \quad (19)$$

where m is the airfoil mass per unit length, m_f is the fixture mass per unit length connecting the airfoil to the plunge springs, x_α is the chord-wise offset from the reference point to the centroid, b is the semi-chord length, and I_p is the moment of inertia per unit length about the reference point. The potential energy of the system can be given by

$$U = \frac{1}{2} k_h h^2 + \frac{1}{2} k_\alpha \alpha^2 - \frac{1}{2} \frac{\theta}{l} hv \quad (20)$$

where k_h is the stiffness per unit length associated with the plunge degree of freedom, k_α is the stiffness in pitch degree of freedom, θ is the electromechanical coupling term, l is the span length of the cantilever, and v is the voltage. The internal electrical energy is

$$W_{ie} = \frac{1}{2} \frac{C_p^{eq}}{l} v^2 + \frac{1}{2} \frac{\theta}{l} hv \quad (21)$$

where C_p^{eq} is the equivalent capacitance of the piezoelectric material.

The virtual work components can be determined as

$$\delta W_{nce} = \frac{Q}{l} \delta v \quad (22)$$

$$\delta W_{nca} = -L\delta h + M\delta\alpha \quad (23)$$

$$\delta W_{sd} = -d_h \dot{h} \delta h - d_\alpha \dot{\alpha} \delta\alpha \quad (24)$$

where Q is the output of the electrical charge, L is the aerodynamic lift per unit length, $M$is the aerodynamic pitching moment per unit length, d_h is the structural damping coefficient per unit length for the plunge DOF and d_α is the structural damping coefficient per unit length for the pitch DOF.

By assuming a harmonic response, it defines $h = \bar{h} e^{j\omega t}$, $\alpha = \bar{\alpha} e^{j\omega t}$, $v = \bar{v} e^{j\omega t}$, $L = \bar{L} e^{j\omega t}$, $M = \bar{M} e^{j\omega t}$. Using the Lagrange equations with the harmonic response assumption, the Equations (19)-(24) will lead to the complex eigenvalue problem below [31]

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{\bar{h}}{b} \\ \bar{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (25)$$

where

$$a_{11} = \left[\beta + \frac{l_h}{\mu} - \kappa(\omega) - \sigma^2 (1 + j\gamma_h) \lambda \right],$$

$$a_{21} = \left(x_\alpha + \frac{l_a}{\mu} \right),$$

$$a_{12} = \left(x_a + \frac{m_h}{\mu} \right),$$

$$a_{22} = \left[r^2 + \frac{m_\alpha}{\mu} - r^2 (1 + j\gamma_\alpha) \lambda \right],$$

and, λ, σ, r, μ, and β are dimensionless terms denoting complex eigenvalue ($\lambda = (\omega_\alpha/\omega)^2$), frequency ratio ($\sigma = \omega_h/\omega_\alpha$), radius of gyration ($r = \sqrt{I_p/m b^2}$), ratio of the airfoil to the affected air mass ($\mu = m/\pi \rho_\infty b^2$), and mass ratio ($\beta = (m + m_f)/m$), respectively, where $\omega_h = \sqrt{k_h/m}$, $\omega_\alpha = \sqrt{k_\alpha/I_P}$, ρ_∞ is the free-stream air mass density, $\gamma_h = d_h\omega/k_h$, $\gamma_\alpha = d_\alpha\omega/k_\alpha$, and l_h, l_α, m_h , m_α are the components of the aerodynamic loads, given by

$$l_h = 1 - j\frac{2}{k}C(k) \quad (26)$$

$$l_\alpha = -a - j\frac{1}{k} - \frac{2}{k^2}C(k) - j\frac{2}{k}\left(\frac{1}{2} - a\right)C(k) \quad (27)$$

$$m_h = -a + j\frac{2}{k}\left(\frac{1}{2} + a\right)C(k) \quad (28)$$

$$m_a = \frac{1}{8} + a^2 - j\frac{1}{k}\left(\frac{1}{2} - a\right) + \frac{2}{k^2}\left(\frac{1}{2} + a\right)C(k) + j\frac{2}{k}\left(\frac{1}{4} - a^2\right)C(k) \quad (29)$$

where α is the dimensionless location of the reference point, k is the reduced frequency, $C(k)$ is the The odorsen function, and $H_n^{(2)}(k)$ are Hankel functions, which are given by

$$k = \frac{b\omega}{U} \quad (30)$$

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + j H_0^{(2)}(k)} \quad (31)$$

$$H_n^{(2)}(k) = J_n(k) - j Y_n(k) \quad (32)$$

$J_n(k)$ is the Bessel function of the first kind, and $Y_n(k)$ are Bessel function of the second kind.

The term $\kappa(\omega)$ is

$$\kappa(\omega) = \frac{j\theta^2}{\omega m l\left(j\omega C_p^{eq} + \frac{1}{R_l}\right)} \quad (33)$$

where R_l is the external electrical load resistance and $\dot{Q} = v/R_l$.

Iterative eigenvalue procedure is employed (e.g. using the P-K Method) to obtain the eigensolution of (25) for $\bar{h}$ and $\bar{\alpha}$. Consequently $\bar{v}$ is determined by

$$\bar{v} = \frac{-j\omega\theta \bar{h}}{j\omega C_p^{eq} + \frac{1}{R_l}} \quad (34)$$

The amount of electrical power can be calculatedby $\bar{v}^2/R_l$.

The two-degree-of-freedom lumped parameter model in this section is extended to the bistable piezoelectric continuous model in the next section.

4 Bistable Piezoelectric Flutter

The flutter analysis, presented in Section 3, is now incorporated in the continuous model of the bistable piezoelectric laminate, that was discussed in Section 2.

A piezoelectric energy conversion mechanism is shown in Fig. 2, where the excitation of the bonded piezoelectric material, on the bistable laminate, is due to aeroelastic flutter.

The bistable piezoelectric system is shown in Fig. 3, which can provide large deflections when subjected to external loading due to the bistable snap-through characteristics of the bistable laminate, as in Fig. 4.

The substructure (bistable laminate), and the piezoelectric strain energies in Equation (17), can be given by

$$U^s = U^{m,s} + U^{th,s} = \frac{1}{2}\int_{A^s}(\bar{N}^T - \bar{N}^{th\,T})\,\bar{\varepsilon} d A^s \quad (35)$$

$$U^p = U^{m,p} + U^{e,p} = \frac{1}{2}\int_{A^s}\bar{N}^T\,\bar{\varepsilon} d A^p \quad (36)$$

By substituting $\bar{N}$ from Equation (9) it obtains,

$$U^s = U^{m,s} + U^{th,s} = \frac{1}{2}\int_{A^s}(Q\,\bar{\varepsilon}^L + Q\,\bar{\varepsilon}^{NL} - Q\alpha\Delta T)^T(\bar{\varepsilon}^L + \bar{\varepsilon}^{NL})\,d A^s \quad (37)$$

$$U^p = U^{m,p} + U^{e,p} = \frac{1}{2}\int_{A^s}(Q\,\bar{\varepsilon}^L + Q\,\bar{\varepsilon}^{NL} - t^p e' E_3)^T(\bar{\varepsilon}^L + \bar{\varepsilon}^{NL})\,d A^p \quad (38)$$

The variation of the strain energy of the system can be expressed by

$$\delta U = \frac{\partial U}{\partial X}\delta X + \frac{\partial U}{\partial v}\delta v \quad (39)$$

where X is the in-plane deformations and curvatures vector and v is the voltage.

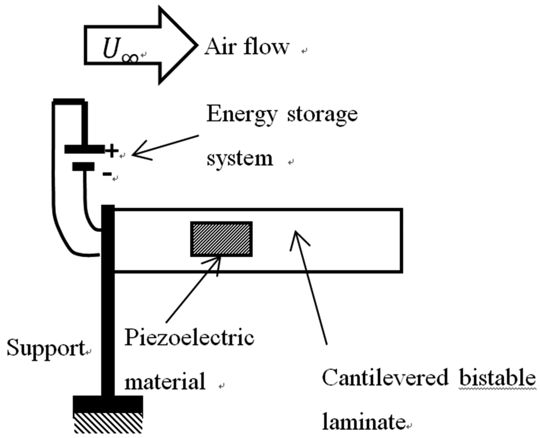

Fig. 2 Piezoelectric flutter energy harvesting.

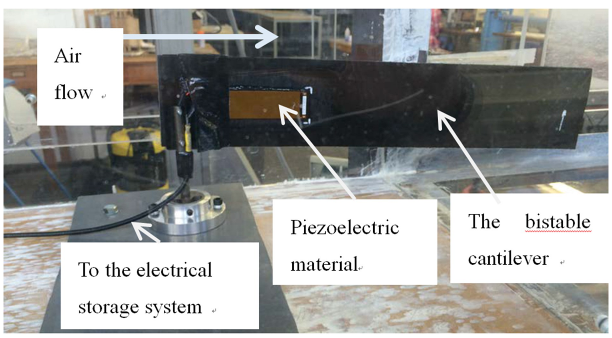

Fig. 3 The bistable piezoelectric system.

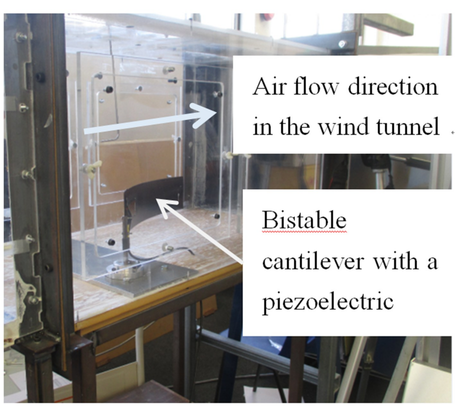

Fig. 4 Demonstration of bistable large deflection.

By combining all the energy terms, the governing dynamic equations can be found, where the energy terms are: the mechanical piezoelectric strain energy, electrical strain energy, mechanical substructure strain energy, thermal strain energy, piezoelectric internal electrical energy, kinetic energy of the system, and non-conservative work. The governing dynamic equations, can be given by the Hamilton's principle in Equation (16) as multipliers of the virtual in-plane deformation and the curvature coefficients (δX) and the virtual voltage (δv). By factorizing δX and δv, their multipliers should be zero in the time interval t_1 to t_2, and the governing equations are derived as follows:

$$\begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{d} \\ \ddot{a} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} d \\ a \end{bmatrix} + \begin{bmatrix} \theta_d \\ -\theta_a \end{bmatrix} v = \begin{bmatrix} L_{aero} \\ M_{aero} \end{bmatrix} \tag{40}$$

$$-\theta_d \dot{d} + \theta_a \dot{a} + \frac{C_p \dot{v}}{t^p} + \frac{v}{R} = 0 \tag{41}$$

where d and a are the in-plane strain coefficient vector and the curvatures, respectively, M is the inertial mass matrix, J denotes the moment of inertia, and L_{aero}, and M_{aero} are the aerodynamic loads. The aerodynamic loads for a 2DOF problem are given in Equations (26) to (29).

$$K_{11} = \int_{A^s} \bar{\varphi}^T A^s \bar{\varphi} d A^s + \int_{A^p} \bar{\varphi}^T A^p \bar{\varphi} d A^p \tag{42}$$

$$K_{12}(a) = \int_{A^s} -a^T B^s \bar{\varphi} + f^T A^s \bar{\varphi} - \frac{1}{2} \Delta T \bar{\alpha} (A^s + B^s) d A^s + \int_{A^p} -a^T B^p \bar{\varphi} + f^T A^p \bar{\varphi} d A^p \tag{43}$$

$$K_{21}(a) = \int_{A^s} -\bar{\varphi}^T B^s a + \bar{\varphi}^T A^s f_{,a} d A^s + \int_{A^p} -\bar{\varphi}^T B^p a + \bar{\varphi}^T f_{,a} a d A^p \tag{44}$$

$$K_{22}(a) = \int_{A^s} a^T D^s - f^T B^s - a^T B^s f_{,a} + f^T A^s f_{,a} d A^s + \int_{A^P} a^T D^p - f^T B^p - a^T B^p f_{,a} + f^T A^p f_{,a} d A^p - \frac{1}{2} \int_{A^s} \Delta T \bar{\alpha} ((A^s + B^s) f_{,a} - (B^s + D^s)) d A^s \tag{45}$$

where $f_{,a}$ denotes $\frac{\partial f}{\partial a}$.

5 Uncertainty quantification for the piezoelectric bistable flutter model

The unavoidable uncertainties in the material, loading, and efficiency of a complex system call for a probabilistic approach. Hence, this paper offers a formulation that considers uncertainty bounds in obtaining a realistic model. Optimal Uncertainty Quantification (OUQ) is used in this paper, which takes into account uncertainty measures with optimal bounds and incomplete information about the system, as a well-defined optimization problem according to probability maximization, subjected to the imposed constraints. The OUQ allows the inspection of the solution for a span of uncertain input parameters, as a reliable and realistic model. The analytical energy balance of the bistable piezoelectric system is given by the Equations (35) through (39). The strain energy of the system for the system, while considering uncertainties and the corresponding optimal bounds, can be given by OUQ. The parameters with uncertainties can include external loading, material characteristics of the bistable laminate and the piezoelectric material, boundary conditions, and the overall electrical circuit efficiency. Therefore, the corresponding parameters that are potentially uncertain in Equations (40) to (45) can be formulated by bounded probabilities.

If the output power of the piezoelectric system is givenby P_o, then it can define $P_o : X \to \mathbb{R}$, $X \to P_o(X)$, with the probability of $\mathbb{P} \in M(X)$, where X and X denote the deterministic and the stochastic forms of the inputs, respectively. In a stochastic representation of a system, it is required that the probability of the power consumption function, $P_o(X)$ to be obtained for a bounded region of the uncertain parameters. If the system is required to supply a certain amount of desired power output, P_d, for instance, in achieving a self-powered scheme as an objective of the problem, failing of the system (which corresponds to $P_o < P_d$) can be formulated as

$$\mathbb{P}[P_d(X) \geq P_o] \leq \epsilon \tag{46}$$

which is referred to the scenario that: 'the probability that the desired output power, P_d, is larger than the produced output power by the piezoelectric system, must be less than a defined $\in$, in order for the system not to fail as a self-powered application. Suppose that the probability function is a member of admissible extremal scenarios A (or $(P_o, \mathbb{P}) \in A$), where A is defined as

$$A : \left\{ \begin{array}{c} p_o : X_1 \times \cdots \times X_n \to \mathbb{R} \\ (p_o, \mu) \mid \mu = \mu_1 \otimes \mu_2 \otimes \cdots \otimes \mu_n \\ \mathbb{E}_\mu [p_o] \leq P_{max} \end{array} \right\} \tag{47}$$

and n denotes the number of uncertain parameters to be considered in the analysis of the power/energy, for parameters defined as inputs to the power/energy function, and these inputs are denoted by X_i, $i = 1, \cdots, n$. Also, $\mathbb{E}_\mu [p_o]$ is the bounded mean output power, μ_i is the probability measure of the input parameter X_i ($\mu_i \in P(X_i)$), and p_o is a possible output function of P_o associated with the corresponding inputs/parameters X_i. The original problem entails optimization over a collection of (p_o, μ) that could be $(P_o, \mathbb{P})$. P_{max} is the output upper bound of the piezoelectric physical limits in producing power/energy. X_i and μ_i parameters can be constrained values with corresponding lower and upper bounds for each i. The optimal bounds on the probability of power consumption according to the required performance can be described by the upper bound, $U(A)$ as

$$U(A) := \sup_{(p_o, \mu) \in A} \mu [p_d(X) \geq P_o],$$

The lower bound, $L(A)$, corresponding to the minimum required power consumption can be stated as

$$L(A) := \inf_{(p_o, \mu) \in A} \mu [p_d(X) \geq P_o]$$

Therefore, the optimal bounds of the problem can be given by

$$L(A) \leq \mathbb{P}[P_d(X) \geq P_o] \leq U(A)$$

The power output can be determined by solving the constrained optimization problem for extremal scenarios A, with the inputs/parameters are also constrained.

If any of the parameters that are used in obtaining the piezoelectric energy/power output, are considered for uncertainty analysis, the upper bound and lower bound for such parameters are introduced, and the problem is solved accordingly, as an optimization problem with the bounds introduced as the problem constraints. For instance, parameters, v, M, J, L_{aero}, and/or M_{aero}, which are responsible in obtaining the output power/energy, can be introduced with well-defined upper and lower bounds in order to span the probability of such parameters being regarded as uncertain or incomplete information about the system/parameter. Then, in a mathematical form we can write, for instance,

$$M \in X_1 := [M_{min}, M_{max}]$$
$$J \in X_2 := [J_{min}, J_{max}]$$
$$L_{aero} \in X_3 := [L_{aero\,min}, L_{aero\,max}]$$
$$M_{aero} \in X_4 := [M_{aero\,min}, M_{aero\,max}]$$
$$v \in X_5 := [v_{min}, v_{max}]$$

In such case, the admissible set A may be given by

$$A: \left\{ (p_o, \mu) \mid \begin{array}{c} p_o : X_1 \times X_2 \times \cdots \times X_5 \to \mathbb{R} \\ \mu = \mu_1 \otimes \mu_2 \otimes \mu_3 \otimes \mu_4 \otimes \mu_5 \\ \mathbb{E}_\mu [p_o] \le P_{max} \end{array} \right\} \tag{48}$$

As seen in this example, the optimization cost function includes multiple parameters. Consequently the problem of the output energy of the piezoelectric system is analyzed for the "worst case" scenario.

If (experimental) sample data is available, the OUQ approach can be extended to the Machine Wald [34] technique, which is equivalent to performing Bayesian inference but optimizing the prior value. In Machine Wald, if an estimation of a function $\Phi(\mu)$, is function θ of sample data d, then the estimation error $\theta(d) - \Phi(\mu)$ is required to be equal to zero.

6 Conclusion

The unavoidable uncertainties in the material, loading, boundary conditions, and system efficiency of the complex bistable piezoelectric flutter energy harvesting model call for a probabilistic approach. In this paper, a realistic model of the bistable piezoelectric system which takes into account the uncertainties in the model with bounded constrained was proposed, as a well-defined problem. This approach allowed obtaining a solution for the bounded output energy in accordance with the bounded inputs and system characteristics, using Optimal Uncertainty Quantification. The OUQ approach was recommended as a more reliable method in estimating the output energy of the system rather than relying merely on the deterministic approach. The future work of this research will involve comparing the experimental results with the numerical solutions of the corresponding OUQ-based model.

References

[1] Khoshnoud, F., McKerns, M. M., de Silva, C. W., Esat, I. I., and Owhadi, H. (2017). Self-powered Dynamic Systems in the framework of Optimal Uncertainty Quantification, ASME *Journal of Dynamic Systems, Measurement, and Control*, Volume 139, Issue 9, 091005-091005-13. doi:10.1115/1.4036367.

[2] De Silva, C. W., Khoshnoud, F., Li, M., andHalgamuge, S. K. (editors), (2015). Mechatronics: Fundamentals and Applications, Chapter 12-Self-powered and Biologically Inspired Dynamic Systems, *Taylor & Francis / CRC Press*, Boca Raton, Florida, USA.

[3] Khoshnoud, F., McKerns, M. M., de Silva, C. W., Esat, I. I., Bonser, R. H. C., and Owhadi, H. (2016). Self-powered and Bio-inspired Dynamic Systems: Research and Education, *ASME International Mechanical Engineering Congress and Exposition*, Phoenix, Arizona, USA, November 11 – 17.

[4] Khoshnoud, F., Dell, D. J., Chen, Y. K., Calay, R. K., de Silva, C. W., and Owhadi, H. (2013). Self-Powered Dynamic Systems, *European Conference for Aeronautics and Space Sciences*, Munich, Germany, Paper 275.

[5] Khoshnoud, F., Zhang, Y., Shimura, R., Shahba, A., Jin, G., Pissanidis, G., Chen, Y. K., and de Silva, C. W. (2015). Energy regeneration from suspension dynamic modes and self-powered actuation, *IEEE/ASME transaction on Mechatronics*, Volume: 20, Issue: 5, pp. 2513 – 2524.

[6] Khoshnoud, F., Lu, J., Zhang, Y., Folkson, R., and de Silva, C. W. (2014). Suspension energy re-

generation for random excitations and self-powered actuation, *IEEE International Conference on Systems, Man, and Cybernetics*, San Diego, CA, USA, pp. 2549-2554.

[7] Khoshnoud, F. de Silva, C. W. (Edited by Vantsevich, V. ndV., and Blundell, M. V.) (2015), Advanced Autonomous Vehicle Design for Severe Environments, (Chapter 8: Mechatronics issues of vehicle control and self-powered systems), *IOS Press*, Fairfax, VA, USA.

[8] Khoshnoud, F., Sundar, D. B., Badi, N. M., Chen, Y. K., Calay, R. K., and de Silva, C. W. (2013). Energy harvesting from suspension systems using regenerative force actuators, *International Journal of Vehicle Noise and Vibration*, Vol. 9, Nos. 3/4, pp. 294 – 311.

[9] Bazios, P., Khoshnoud, F., and Esat, I. I. (2017). Energy harvesting from suspension system and self-powered vibration control for a seven degree of freedom vehicle model, *Journal of Multi-body Dynamics*, Proceedings of the Institution of Mechanical Engineers, dio: 10.1177/1464419317735990, 2017.

[10] Khoshnoud F. and de Silva, C. W. (2012) Recent advances in MEMS sensor technology - Mechanical Applications, *IEEE Instrumentation and Measurement*, Volume 15, Issue 2, pp. 14 – 24.

[11] Raje, H., Hawkes, L., and Yuen, P., 2015, "Piezoelectric energy harvesting using bistable material," MSc dissertation, Brunel University London, Uxbridge, UK.

[12] Khoshnoud, F., Owhadi, H., de Silva, C. W., Zhu, W., and Ventura, C. E. (2011). Energy harvesting from ambient vibration with a nanotube based oscillator for remote vibration monitoring, *Proc. of the Canadian Congress of Applied Mechanics*, Vancouver, BC.

[13] Khoshnoud, F., Esat, I. I., de Silva, C. W., Rhodes, J. D., Kiessling, A. A., and Quadrelli, M. B., Self-powered Solar Aerial Vehicles: Towards Infinite Endurance UAVs, submitted.

[14] Ju, S.H., Lin, H.T., and Huang, J.Y. (2009). Dominant frequencies of train-induced vibrations, Journal of Sound and Vibration, 319(1-2), 247-259.

[15] Wood, O. J., Featherston, C. A., Kennedy, D., Eaton, M., and Pullin, R. (2012). Optimised vibration energy harvesting for aerospace applications, *Key Engineering Materials – Structural Health Monitoring* II, 518, 246-260.

[16] Zhou, S., Cao, J., Erturk, A., and Lin, J. (2013). Enhanced broadband piezoelectric energy harvesting using rotatable magnets, *Applied Physics Letters*, 102 (173901), 1-4.

[17] M. W. Hyer, Calculations of the room-temperature shapes of unsymmetric laminates, Journal of Composite Materials, 15(July), 296-310, 1981.

[18] Arrieta, A. F., Hagedorn, P., Erturk, A., and Inman, D. J. (2010). A piezoelectric bistable plate for nonlinear broadband energy harvesting, Applied Physics Letters, 97(10), 1-3.

[19] Betts, D. N., Kim, H. A., Bowen, C. R., and Inman, D. J. (2012). The optimal configuration for broadband energy harvesting using bistable phenomenon, *Applied Physics Letters*, 100(114104), 1-4.

[20] Betts, D. N., Kim, H. A., and Bowen, C. R. (2012). Preliminary study of optimum piezoelectric crossply composites for energy harvesting, *Smart Materials Research*, 1-8.

[21] Dano, M. L. and Hyer, M. W. (1998). Thermally induced deformation behavior of unsymmetric laminates, *International Journal of Solids and Structures*, 35(17), pp. 2101-2120.

[22] Dano, M. L. and Hyer, M.W. (2003). SMA-induced snap-through of unsymmetric fiber-reinforced composite laminates, *International Journal of Solids and Structures*, 40(22), pp. 5949-5972.

[23] Ren, L. and Parvizi, A. (2006). A model for shape control of cross-ply laminated shells using a piezoelectric actuator, *Journal of Composite Materials*, 40 (14), pp. 1271-1285.

[24] Betts, D. N., Bowen, C. R., Kim, H. A., Guyer, R., Le Bas, P., and Inman, D. J. (2014). Modeling the dynamic response of bistable composite plates for energy harvesting, 55*th AIAA Structures, Strucutral Dynamics, and Material Conference*.

[25] Tawfik, S., Dancila, D. S., and Armanios, E. (2011). Planform effects upon the bistable response of cross-ply composite shells, *Composites Part A: Applied Science and Manufacturing*, 42(7), pp. 825-833.

[26] Arrieta, A. F., Bilgen, O., Friswell, M. I., and Ermanni, P. (2013). Modelling and configuration control of wing-shaped bi-stable piezoelectric composites

under aerodynamic loads, Aerospace Science and Technology, 29(1), pp. 453-461.

[27] Bowen, C. R. Nonlinear Dynamic Analysis of Piezoelectric Bistable Laminates, Personal communications.

[28] Tavakkoli, S. M., Weaver, P. M., Bowen, C. R., Inman, D. J., and Kim, H. A. (2015). An analytical study on piezoelectric-bistable laminated with arbitrary shapes for energy harvesting, 7th *ECCOMAS Thematic Conference on Smart Structures and Materials SMART*.

[29] Owhadi, H., Scovel, C., Sullivan, T., McKerns M., and Ortiz, M. (2013). Optimal Uncertainty Quantification, *SIAM Review*, Vol. 55, No. 2, pp. 271-345.

[30] Dano, M. L. and Hyer, M. W. (1998). Thermally-induced deformation behavior of unsymmetric laminates, *Int. J. solids Structures* 35, 17, pp. 2101-2120.

[31] Erturk, A. and D. J. Inman, (2011). Piezoelectric Energy Harvesting," *John Wiley & Sons*.

[32] Arrieta, A., Hagedorn, P., Eturk, A., and Inman, D. (2010). A Piezoelectric Bistable Plate For Nonlinear Broadband Energy Harvesting, *Applied Physics Letters*, 97.

[33] DeMarqui Jr, C., Erturk A., and Inman, D. J. (2010), Piezoaeroelastic Modelling and Analysis of a Generator Wing with Continuous and Segmented Electrodes, *Journal of Intelligent Material Systems and Structures*, Online Vol 21 (10).

[34] Owhadi, H. and Scovel, C. (2015). Towards Machine Wald, Submitted on 10 Aug 2015, last revised 1 Oct 2015, http://arxiv.org/abs/1508.02449 (Last accessed Feb. 2019).

Authors' Biographies

Farbod Khoshnoud, PhD, PGCE, CEng, M. IMechE, M. ASME, HEA Fellow, is a faculty member at California State Polytechnic University, Pomona, and a Research Affiliate at the NASA Jet Propulsion Laboratory, Caltech. His current research areas include Self-powered and Biologically Inspired Dynamic Systems, Unmanned Systems (e.g. collaborative and networks), and Experimental Quantum Collaborative Robots. He was an Associate Professor of Mechanical Engineering at California State University, USA, a visiting Associate Professor in the Department of Mechanical Engineering at the University of British Columbia (UBC), Vancouver, Canada, in 2017, a Lecturer in the Department of Mechanical Engineering at Brunel University London, UK, 2014-16, a senior lecturer at the University of Hertfordshire, 2011-2014, a visiting scientist and postdoctoral researcher in the Industrial Automation Laboratory, Department of Mechanical Engineering, at UBC, Vancouver, 2007-2012, a visiting researcher at California Institute of Technology, USA, 2009-2011. He carried out postdoctoral research in the Department of Civil Engineering at UBC, 2005-2007. He received his Ph.D. in Mechanical Engineering from Brunel University in 2005. He has worked in industry as a mechanical engineer for over six years. He is an associate editor of the *Journal of Mechatronic Systems and Control* (formerly *Control and Intelligent Systems*).

Christopher Rhys Bowen has a BSc degree in Materials Science from the University of Bath (1986-1990) and a DPhil in Ceramics from the University of Oxford (1990-1993). His post-doctoral work has been undertaken at Tecnische Universität Harburg-Hamburg and University of Leeds (1994-1996). He was Senior Scientist at the Defence Evaluation and Research Agency from 1996-1998. He joined the University of Bath as a Lecturer in 1998 and is now Professor of Materials. His research areas include energy harvesting, piezoelectric materials, dielectrics and functional ceramics.

Dr. Cristinel Mares is a Reader in the Mechanical and Aerospace Engineering department at Brunel University London, UK.