Reduced-order design of Robust H∞ Controller for an Inertial Stabilized Aerial Platform

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Abstract: The uncertainty disturbance is one of the main disturbances that seriously influences the stabilization precision of an aerial inertially stabilized platform (ISP) system. In this paper, to improve the stabilization precision of the ISP under disturbance uncertainty, a robust H∞ controller is designed in this paper. Then, the reduction order is carried out for high-order controllers generated by the robust H∞ loop shaping control method. The application of the minimum implementation and balanced truncation algorithm in controller reduction is discussed. First, the principle of reduced order of minimum implementation and balanced truncation are analyzed. Then, the method is used to reduce the order of the high-order robust H∞ loop shaping controller. Finally, the method is analyzed and verified by the simulations and experiments. The results show that by the reduced-order method of minimum implementation and balanced truncation, the stabilization precision of the robust H∞ loop shaping controller is increased by about 10%.

Key words: Aerial Remote Sensing; Inertial Stabilization Platform; Robust H∞ Control; Controller Reduction; Balanced Truncation

1 Introduction

Inertial stability platform (ISP), which supports and stabilizes the line of sight (LOS) of the imaging sensors, is one of crucial components of an aerial remote sensing system. The stabilization precision of the ISP has a great effect on the imaging quality. The uncertainty disturbance on model parameters is one key factorthat affects the stabilization precision of the ISP control. For uncertain systems, it is difficult to obtain an accurate model of the control object.

Robust H∞ method considers the influence of system uncertainty that not only guarantees the robust stability of the control system, but also optimizes certain performance indicators. Robust H∞ control particularly concerns analysis and processing of nonlinear systems with unmodeled dynamics. It provides a design method for a robust controller in the frequency domain for the control systems with model perturbations. It combines the advantages of classical design theory of frequency domain and modern control theory of state space. By implementing loop shaping design in the frequency domain, the robust control of system is realized. Under the influences of system uncertainty on the control, robust H∞ control method can not only realize the robust stability of the system, but also improve the control performance. The use of the state space method has obvious advantages of accurate calculation and optimization that are possible in the time domain method.

However, the resulting order of the robust controller can be generally higher than that of the controlled object, which is even more than three times in some cases. Then, the extensive coupling problem and high computation problem will occur. Clearly, a controller of low order is more convenient to program. If the designed controller is linear, it will reduce the computational complexity, thus improving the reliability of the entire control system. For a high-order robust H∞ loop shaping controller, the method of minimum implementation and balanced truncation may be used to reduce the order and perform simulation analysis.

Reference [1] describes in detail the step-down method of balanced interception. In this system, the aeroengine controller is the reduced order object. Reference [2] uses balanced truncation to reduce the order of the controller. Reference [3] uses the frequency-weighted right-level decomposition method to reduce the order. In [4], a robust H∞ controller is designed, and in view of the high order of the system controller, the minimum information loss method is used to reduce the order. In [5], several methods of reducing the order are studied, and the order is reduced on the H∞ controller.

In the present paper, to suppress the influences of uncertain disturbances on the stabilization precision of the ISP, the order reduction is carried out for high-order controllers generated by the robust H∞ loop shaping control method. The developed method is analyzed and verified by simulations and experiments.

2 Background

2.1 Inertial stabilizing aerial platform

The basic structure of the three-axis ISP is shown in Fig.1.

The structure of the three-axis ISP consists of three frames. Specifically, from the outside to the inside, there are the roll frame, the pitch frame and the azimuth frame [6]. The rotary axis of the horizontal frame is along the flight direction of the aircraft, to isolate the roll angle motion of the aircraft; the rotary axis of the pitch frame is along the direction of the aircraft wing, to isolate the pitch angle motion of the aircraft; the rotary axis of the azimuth frame is vertically downward, to isolate the azimuth movement of the aircraft.

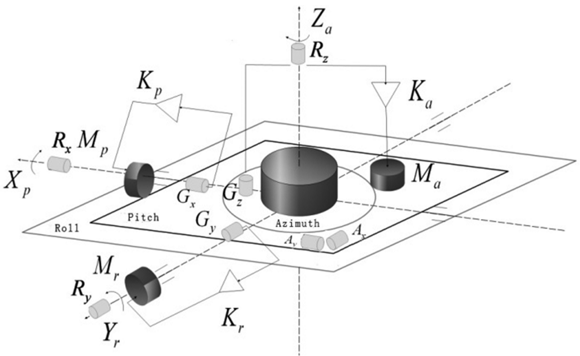


Fig. 1 Schematic diagram of three-axis inertial stabilization platform system.

Based on the system modeling of the three-axis inertial stabilization platform[7], the servo control system adopts a three-loop control structure composed of a tracking loop (position loop), a stable loop (rate loop) and a current loop (current loop), as shown in Fig.2.

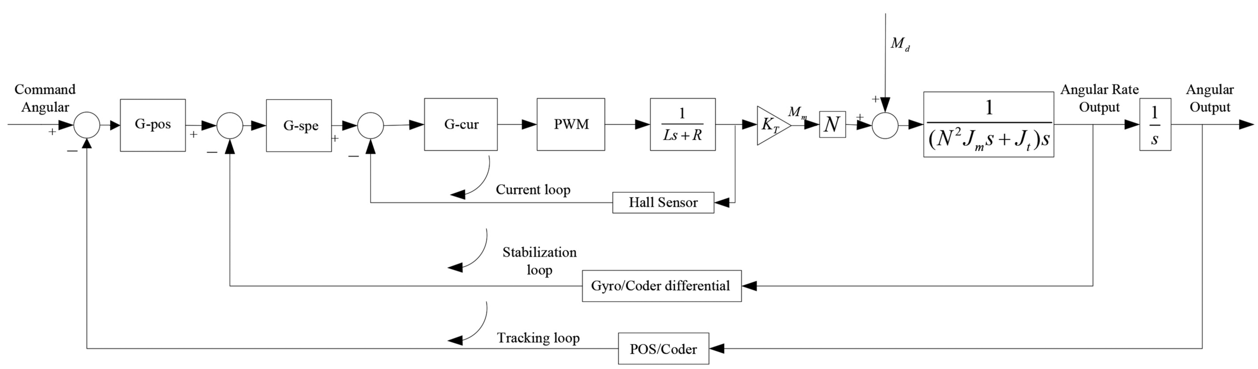


Fig. 2 A block diagram of traditional three-loop control system for ISP.

When the ISP is operating, the Position and Orientation System (POS), which is fixed with the camera, provides high-precision angular position tracking reference in real time for the position loop of the ISP position loop, to achieve stable tracking. Three rate-gyro sensors are installed on the three frames to measure the spatial rotational angular velocities of each frame in real time, which are used as feedback to compensate for the disturbances in the stable loop.

3 Reduced-order Design of Robust H∞ Controller

3.1 Minimum implementation of the transfer function

State space method uses a set of 1st order differential equations to describe the state of a control system or the controlled object. In the control system, suppose that there are m input variables, $u_1, u_2, \cdots, u_m$; there are r output variables $y_1, y_2, \cdots, y_r$; and there are n state variable $x_1, x_2, \cdots, x_n$; The controlled object can be described by the following equations[8]:

$$\frac{dx_j(t)}{dt} = f_j(t, x_1(t), x_2(t), \cdots, x_n(t), u_1(t), u_2(t), \cdots,, u_m(t))\ j = 1,2,\cdots,n \tag{1}$$

$$y_k(t) = g_k(t, x_1(t), x_2(t), \cdots, x_n(t), u_1(t), u_2(t), \cdots,, u_m(t))\ k = 1,2,\cdots,r \tag{2}$$

Equation (1) is the set of state equations, equation (2) is the set of output equations, and f_j and g_k are functions that are determined according to the characteristics of the controlled object and that satisfy certain special conditions.

Since the separated state distribution is arranged into the components of the matrix at the time of design, it is inevitably expressed in the state space matrix of the system (i.e., the system matrix). In this way, there are both states that have a great influence on the output and many unnecessary states [9]. So, the state space matrix of the control system becomes of high dimensionality. Therefore, it is needed in the construction process to remove unnecessary states and to obtain the minimum state space after design.

According to the given transfer function matrix, the first step is to write the satisfied controllable implementation, and the second step is to find the observable subsystem; or the first step is to write the satisfied observable implementation, and the second step is to determine the control subsystem. Both methods can achieve a minimum implementation [10].

Because balanced truncation is required on a minimal implementation basis, the minimum implementation is done in preparation for the balanced truncation of the controller. In actual engineering design, the minimum implementation process removes the uncontrolled or unobservable state of the controlled system controller, and the balanced truncation method discards the state part of the weak control of the controlled system. At the same time, it abandons the state part of the weak dynamics.

3.2 Reducing the order of the controller with balanced truncation

For second-order systems:

$$\begin{cases} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = Bu(t) \\ y(t) = C_1 q(t) + C_2 \dot{q}(t) \end{cases} \tag{3}$$

where $u(t) \in R^m$, $y(t) \in R^p$, $q(t) \in R^n$, $B \in R^{n\times m}$, $C_1, C_2 \in R^{p\times n}$, $M, D, K \in R^{n\times n}$. Assume that M is reversible, $q(t)$ is the position, $\dot{q}(t)$ is the speed, $u(t)$ is the external input, B is the input distribution matrix, and $y(t)$ is output vector. C_1, C_2 are the output observation matrices.

The transfer function of system (3) is [11]:

$$G(s) = (sC_2 + C_1)P(s)^{-1}B \tag{4}$$

where

$$P(s) = (s^2 M + sD + K) \tag{5}$$

Equation (5) is a characteristic polynomial matrix. The poles in the model reduction play a very important role; for example, when all the poles of the controlled system are in the left half plane, the system is stable. If at least one pole is on the right half plane, the system is unstable.

In the design and application of an ISP control system, the order of the robust controller will be higher than the order of the controlled object. In some cases, it will even reach more than 3 times the order of the controlled object, which leads to a significant increase of the tasks of analysis, simulation and design, and makes it impossible to complete the calculations in a reasonable time. In order to solve the problems often encountered in this practical project, based on maintaining the second-order structure of the controlled system, the designer often wants to construct a $k(k < n)$ -dimensional low-order controlled system.

$$\begin{cases} \hat{M}\ddot{\hat{q}}(t) + \hat{D}\dot{\hat{q}}(t) + \hat{K}\hat{q}(t) = \hat{B}u(t) \\ \hat{y}(t) = \hat{C}_1\hat{q}(t) + \hat{C}_2\dot{\hat{q}}(t) \end{cases} \quad (6)$$

The two matrices associated with the system (6) are, the observable Gramian matrix Q and the controllable Gramian matrix P. They are the only solutions to the following equations [12]:

$$AP + PA^T + BB^T = 0; A^TQ + QA + C^TC = 0 \quad (7)$$

There is a balanced state space implementation $(C_{bal}, A_{bal}, B_{bal})$ such that its Gramian matrix is $\overline{P} = \overline{Q} = \Sigma$, where Σ is a diagonal matrix. The smaller the value of $x_0^T P^{-1} x_0$, with the state x_0, better is the controllability of the system, because the control input for this state can be relatively small. Therefore, when $\overline{P} = \overline{Q} = \Sigma$, the value on the diagonal of the P matrix and Q matrix can very accurately measure the control quantity of the state input of the system and the severity of the influence of this control quantity on the system output [13].

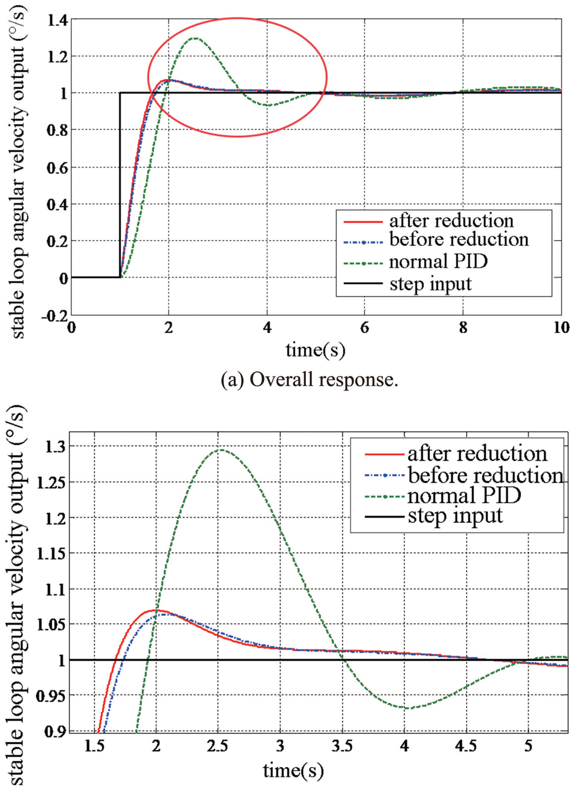
In summary, the basic idea of balanced truncation is to preserve the most controllable and most observable states in the transfer function matrix.

4 Simulation analysis

In MATLAB´s reduced-order operation, first use Minreal to get the minimum implemented controller. Then use the statements of Balreal and Modred in the Application Toolbox, for the reduced order processing of the robust H∞ loop shaping controller.

4.1 Reduced-order design of stable loop controller

The high-order robust H∞ stabilized loop shaping controller reduces the order to obtain a low-order controller, and the reduced-order controller is compared with the controller before the reduction and with conventional proportional-integral-derivative (PID) control.



(a) Overall response.

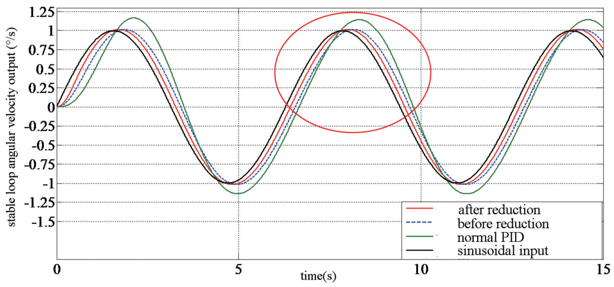
(b) Partial enlargement of response.

Fig. 3 Contrast diagram of robust H∞ loop shaping controller before and after reduced-order of stable loop step response and PID controller.

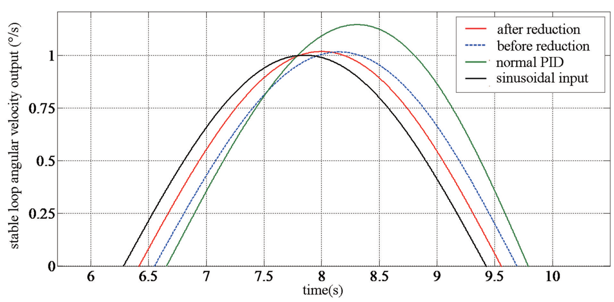
Table 1 Contrast of robust H∞ loop shaping controller before and after reduced-order of stable loop step response and PID control.

	Adjustment time /s	Improved /% than PID	Overshoot°/s	Improved /% than PID
Normal PID	3.4		0.3	
Before reduction	1.85	-45.6	0.06	-80
After reduction	1.75	-48.5	0.07	-76.7

In the stable loop, after adding the uncertain disturbance of inertia parameter and gyro drift, when the input signal is a step signal, the experimental results are summarized in Table 1 and Fig.3. It can be seen from Fig. 3(b) that the angular velocity output adjustment time of the reduced-order robust H∞ stable loop shaping controller is 1.75s, compared to 1.85s before reduction, which is 5.4% shorter than that before reduction; the PID control method is 3.4s, and compared to the PID control it is shortened by 48.5%; the robust H∞ loop shaping controller has a response increase before the step-down, resulting in an overshoot 0.07°/s, and for the PID control method it is 0.3°/s, representing a reduction by 76.7%.



(a) Overall response.



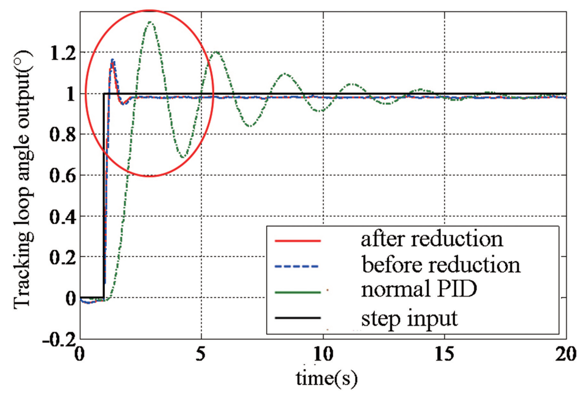
(b) Partial enlargement of response.

Fig. 4　Contrast diagram of robust H∞ loop shaping controller before and after reduced-order of stable loop sinusoidal response and PID control.

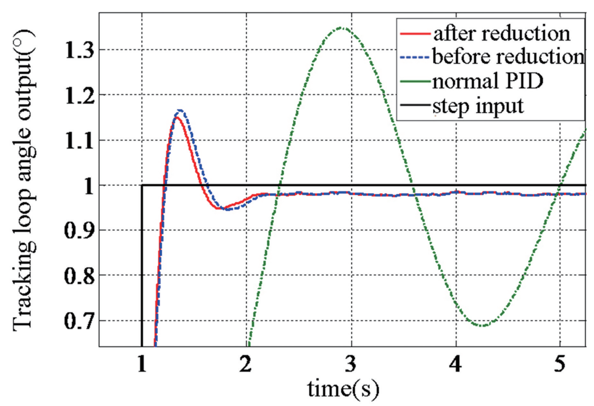
In the stable loop, after adding the uncertain disturbance in the inertia parameter and the gyro drift, when the input signal is sinusoidal, it can be seen from Fig. 4(b) that the reduced-order robust H∞ stable loop shaping controller provides an angular velocity tracking accuracy lower than that before the reduced order, and it is also significantly improved compared to PID control.

4.2　Reduced-order design of tracking loop controller

The high-order robust H∞ tracking loop shaping controller reduces the order to obtain a low-order controller. The reduced-order controller with the controller will be compared before the reduction and the conventional PID.



(a) Overall response.



(b) Partial enlargement response.

Fig. 5　Contrast diagram of robust H∞ loop shaping controller before and after reduced-order of tracking loop step response and PID controller.

Table 2　Contrast table of robust H∞ loop shaping controller before and after reduced-order of tracking loop step response and PID controller.

	Adjustment time /s	Improved /% than PID	Overshoot/°	Improved /% than PID
Normal PID	9.02		0.348	
Before reduction	0.93	-89.7	0.166	-52.3
After reduction	0.8	-91.1	0.150	-56.9

In the tracking loop, the uncertain disturbance was added in the inertia parameter, the gyro drift, add accelerometer noise in the tracking loop. When the input is a step signal, using the robust H∞ controller, the tracking loop angle output is as shown in Table 2 and Fig.5. It can be seen from Fig. 5(a) that the tracking loop angle output adjustment time of the reduced H∞ loop shaping controller is 0.8s, and it is 0.93s before the reduction, which is shortened by 20.0% due to the order reduction. The degree of overshoot of the reduced-order robust H∞ loop shaping controller is 0.150°, and it is 0.166° before the step-down, hence 10.1% shorter than that before reduction; with the PID control method it is 0.348°, hence the comparative reduction is 56.9%.

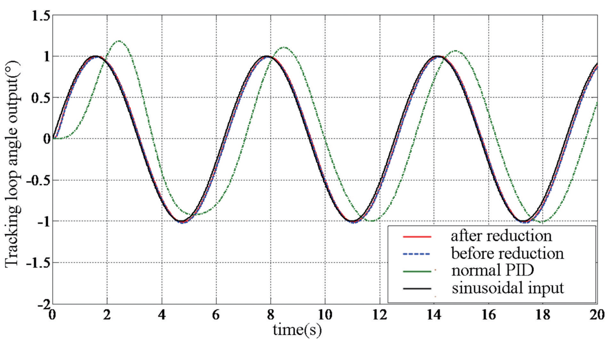


Fig. 6 Contrast diagram of robust H∞ loop shaping controller before and after reduced-order of tracking loop sinusoidal response and PID control.

In the stable loop, we add an uncertain disturbance to the inertia parameter, gyro drift, add accelerometer noise in the tracking loop. When the input signal is sinusoidal, it can be seen from Figure 6 that the tracking loop angle tracking accuracy of the reduced H∞ loop shaping controller after results in a significant improvement compared to the PID control.

It can be seen from the simulation results that the tracking loop robust H∞ controller provides a significant improvement on the model parameter uncertainty disturbance perturbation compared to the pre-step reduction.

5 Experimental Verification and Analysis

Overall performance of the platform was tested under the moving base. The experiment focused on the leveling function. Figure 7 shows view of the experimental equipment and the test conditions.

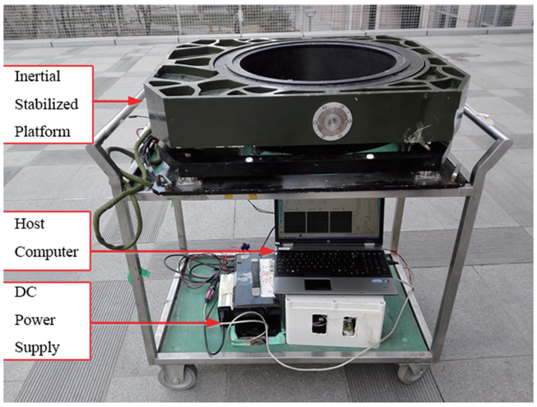


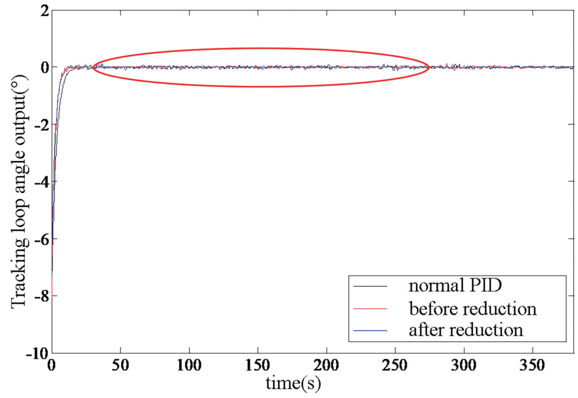
Fig. 7 Robust H∞ loop shaping control platform diagram with moving base.

During the experiment, multiple sets of data were measured. The analysis was performed on one set of the measured experimental data. For the testing, the new section of the main building lobby was selected. The ability to level the platform when the base is disturbed was tested.

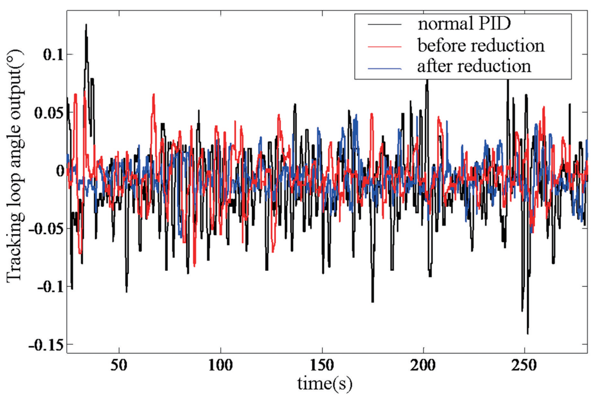
Table 3 Contrast table of dynamic pedestal leveling of robust H∞ loop shaping platform before and after reduced-order and PID control.

	Experiment duration /s	RMS error/°	Improved /% than PID
Normal PID	380	0.0356	
Before reduction	380	0.0287	-19.4
After reduction	380	0.0252	-29.2

Under the moving base, in the stable loop and the tracking loop, the reduced loop H∞ controller was used. The resulting tracking loop angle output is as shown in Table 3 and Fig. 8. It can be seen from Fig. 8(b) that the tracking loop angle output RMS of the reduced-order controller using the robust H∞ loop is 0.0252°, and it is 0.0287° before the reduction. This represents a 12.2% improvement. The angle corresponding to the PID control method is 0.0356°, and it is seen that the reduced controller has resulted in a 29.2% improvement compared with the PID control. The experimental results of the moving base show that the reduced H∞ loop shaping controller with the reduced order has resulted in performance close to that before the reduction, and the suppression ability of the uncertainty of the model parameters is significantly improved compared with the PID control.



(a) Overall response.



(b) Partial enlargement of response.

Fig. 8 Contrast diagram of dynamic pedestal leveling of robust H∞ loop shaping platform before and after reduced-order and PID control.

6 Conclusion

This paper presented a reduced-order design of robust H∞ controller for an inertial stabilized aerial platform. The method of minimum implementation and balanced truncation was used to reduce the order of the high-order robust H∞ loop shaping controller, which was analyzed and verified by both simulation and experimentation. The experimental results of the moving base showed that the robust H∞ loop shaping controller after the reduced order resulted in performance close to that before the reduction, and the suppression ability of the uncertainty disturbance of the model parameters was also significantly improved compared with PID control. Therefore, with the reduced-order method of minimum implementation and balanced truncation, the robust H∞ loop shaping controller was able to reduce the controller order while ensuring good system performance.

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References

[1] Li, Q. and Sun, J. (2008). Model Reduction Method for Aeroengine Controller. *Journal of Nanjing University of Aeronautics & Astronautics*, 40(3), pp. 284-287.

[2] Li, H. and Peng, J. (2006). Experimental study of active structural control with model and controller reduction. *Journal of Vibration Engineering*, 19(1), pp. 17-23.

[3] Huang, C. and Wang, H. (2011). Controller Order-reduction for Full-vehicle Active Suspension System Based on Frequency-weighting Method. *China Mechanical Engineering*, (11), pp. 1366-1369.

[4] Wang, H., Wang, B. and Chen, W. (2012). A Study on the Order Reduction of Vehicle Active Suspension Controller by Using Minimum Information Loss Method. *Automotive Engineering*, 34(11), pp. 995-998.

[5] Anderson, B. D. O. and Liu, Y. (1989). Controller reduction: concepts and approaches. *IEEE Transactions on Automatic Control*, 34(8), pp.802-812.

[6] Masten, M. K. (2008). Inertially stabilized platforms for optical imaging systems. *IEEE Control Systems*, 28(1), pp. 47-64.

[7] Zhou, X., Jia, Y. and Zhao, Q. (2016). Dual-rate-

loop control based on disturbance observer of angular acceleration for a three-axis aerial inertially stabilized platform. *Isa Transactions*, 63, pp. 288-298.

[8] Dullerud, G. E. and Paganini, F. (2000). *A Course in Robust Control Theory: A Convex Approach*. New York: Springer.

[9] Moore, B. (1981). Principal component analysis in linear systems: Controllability, observability, and model reduction*Automatic Control, IEEE Transactions on*, 26(1), pp. 17-32.

[10] DeSchutter, B. (2000). Minimal state-space realization in linear system theory: an overview. *Journal of Computational and Applied Mathematics*, 1(1), pp. 331-354.

[11] Shi, X. and Tang, Y. (2009). Model reduction using balanced truncation. *Journal of Liaoning Normal University (Natural Science Edition)*, 32(4), PP. 422-425.

[12] Tang, Y. (2009). *Model reduction of second-order systems using balanced truncation*. M.D. Dalian University of Technology

[13] Pernebo, L. and Silverman, L. (1982). Model reduction via balanced state space representations. *Automatic Control IEEE Transactions on*, 27(2), pp. 382-387.

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